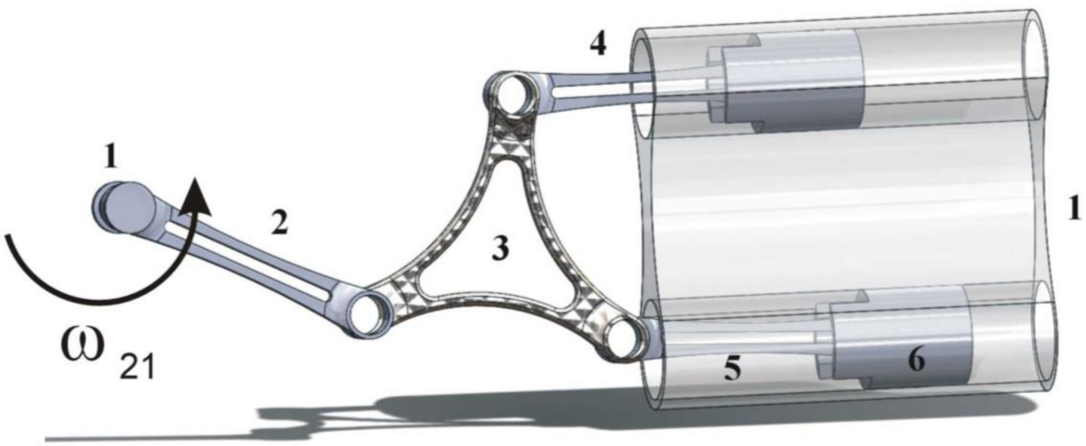
Ján Vavro
Ján Vavro jr.

Analysis and synthesis of planar mechanisms



Authors: prof. Ing. Ján Vavro, CSc., doc. Ing. Ján Vavro, PhD.
Authors workplace: Faculty of Industrial Technologies, Alexander Dubček University of Trenčín,
I. Krasku 491/30, 020 01 Púchov, Slovak Republic, fpt.tnuni.sk
e-mails: jan.vavro@tnuni.sk, jan.vavro.jr@tnuni.sk,

Analysis and synthesis of planar mechanisms

Reviewers:

prof. Ing. Jozef Bocko, CSc. – Technical University of Košice

prof. Dr. Ing. Milan Sága – University of Žilina

prof. Ing. Alžbeta Sapietová, PhD. – University of Žilina

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CONTENT

Introduction

The monograph "Analysis and synthesis of planar mechanisms" deals with the numerical analysis and synthesis of planar mechanisms.

The solution of real problems from practice is often based on the solution of complex systems relating to differential, integral as well as algebraic equations. In the most cases, it is not possible to obtain analytical solutions and therefore, the designers use numerical procedures, using modern computer technology.

The modern computational methods depend on the creation or formation of a virtual model along with the subsequent simulation of the specified system and by this way, it is important to point that the formation and simulation belong to inseparable part of work of current designer in order to solve the whole complex of problems, the solution of which can bring significant economic benefits.

In relation to the requirements of the practice, the main work of designer is to specify and optimise the parameters of the designed device with respect to its weight, shape, geometry or other dynamic properties. The main goal of design work is to save material and to find the best solution from the aspect of the material usage as well as the appropriate shape of the structure.

Nowadays, the increased requirements for material consumption, operating lifetime, durability, reliability of products and machine devices require new progressive approaches to solve the tasks of technical practice. Using the suitable computational or numerical programs, the precise, fast and efficient study can be carried out, while the given study more or less affects the static and dynamic characteristics of the machine or device.

The main aim of the monograph is to acquaint the reader with the fundamental terms of theory for understanding the process of solution of the given tasks and to show the issues, approaches, procedures for formulation of the theory, modeling and simulation of determined solid body systems which can be used as examples of devices or machines from practice.

The content of the monograph is divided into seven chapters. *The first chapter* is devoted to the computational modeling, which allows real systems to be investigated using mathematical relations or equations and there is also the brief description of the appropriate approximation methods of continuum mechanics as well as the most commonly used numerical methods. *The second chapter* is devoted to the introduction to the finite element method (FEM). In the given chapter, the basic principles of the finite element method are elaborated, while the main attention is paid to the four-node tetrahedral space element which was used for modeling of planar solid body systems.

The third chapter is closely connected with kinematic analysis of planar solid systems, the basic concepts and theory and terms which are necessary for creation or formation of planar solid body systems. *The fourth chapter* is based on to the expression of kinematic quantities at rotary motion, general planar motion and simultaneous motion of solid body in the matrix form. *The fifth chapter* deals with the solution of kinematic variables or parameters (quantities), using vector method. Numerical solution of kinematic (parameters) quantities was applied to various solid body systems. *The sixth chapter* includes kinematic and dynamic analysis, and stress distribution in planar solid body systems, using SolidWorks software. Numerical application was carried out for several examples. The last, *the seventh chapter* is devoted to the description of the individual procedures which are necessary for the performance of the kinematic and dynamic analysis of six-item (member) or body system, using SolidWorks software.

I believe that the monograph will be the great contributing aid for academic specialists as well as for designers, investigators and other experts from practice.

At the end, I consider it a nice duty to express many thanks to the reviewers for valuable comments, recommendations and ideas which have led to the enhancement of the given monograph.

Púchov, October 2023 *Authors*

1 Computational Modeling of Planar Solid Body Systems

Static, kinematic and dynamic analysis of planar solid body systems is closely connected with a virtual (mathematical) model of the investigated system, which has to be created or formed as the first in the modeling process. The virtual (mathematical) model describes all the essential properties of the real system with the boundary or critical conditions, such as dimensions, weights, arrangement of individual bodies, kind of interconnection, stiffness or rigidity of individual bodies, way of loading by external forces as well as degrees of freedom.

Computational modeling is based on the similarities between the real and abstract systems and it allow us to investigate the real systems by help of abstract systems, using mathematical relations or equations. The given modeling is suitable for the study of even very complex physical phenomena in extensive systems if we are able to describe the phenomenon as well as the system in a sufficient mathematical way.

Complex physical dynamic phenomena are mostly expressed by simple or partial differential equations or their systems (including additional conditions) and they can be commonly solved in a numerical way, using computational technology. The fundamental principles of the numerical methods are based on the division or transformation (discretization) of dependent variables of physical quantities in the computational domain into the individual, discrete values in the created nodal points of the geometric model with the reference to the issue or task.

Numerical solution of the linear and nonlinear differential equations, which describe particular temperature, velocity, stress, strain filed or displacement fields, is a set of numbers due to which we can get the resultant fields of individual dependent variable physical quantities ($T, w, \delta, \varepsilon, u, \ldots$). Numerical methods are used to solve the dependent variable physical quantities in the finite number of nodes of the discretization mesh in the computational domain.

The common feature of all numerical methods is connected with the effort to get the solution of the differential equation to the level of algebraic equations and to solve these resulting algebraic equations by common matrix calculation. Numerical methods make it possible to obtain a solution of a physical quantity in a specific number of discrete areas (nodes) for the selected differential mesh (finite element mesh) in the whole area or on the surface part of the created geometric model.

The basic numerical method for solving differential equations is the finite element method (FEM). The solution of differential equations is associated with the search for a minimum of the strictly defined functions which are also called shape functions. The principle of FEM is that we substitute corresponding quantity (e.g.: temperature, speed or velocity, etc.) by a discrete model, which is determined by a set of corresponding continuous functions (polynomials) for a finite number of sub-regions (elements).

The elements of the observed area are obtained by dividing the given area into a set of sub-regions of the simple form (triangles, quadrilaterals, tetrahedrons, pyramids). The functions, which are searched, are approximated within the boundaries of each sub-region by the polynomials and by this way, the coefficients of the approximating polynomials are expressed through the values of the searched functions in the finite number of nodal points of the elements. The sub-region with selected determined nodes is called an element. The mutual interaction between finite elements takes place exactly through the nodal points of the elements. The solution of the given task or issue is based on the calculation of the specific numerical values of the searched quantity in nodal points of the model from the system of linear algebraic equations.

Using the computer, the solution of a mathematical model replaces an experiment of a real system and therefore, it is called mathematical experimentation or simulation. In this case, the investigated problem is analysed deterministically (the outputs are introduced with the precision of experimental errors), using the numerical methods, while the model solution is implemented through a suitable program system. The computational modeling scheme is shown in Fig. 1.1.

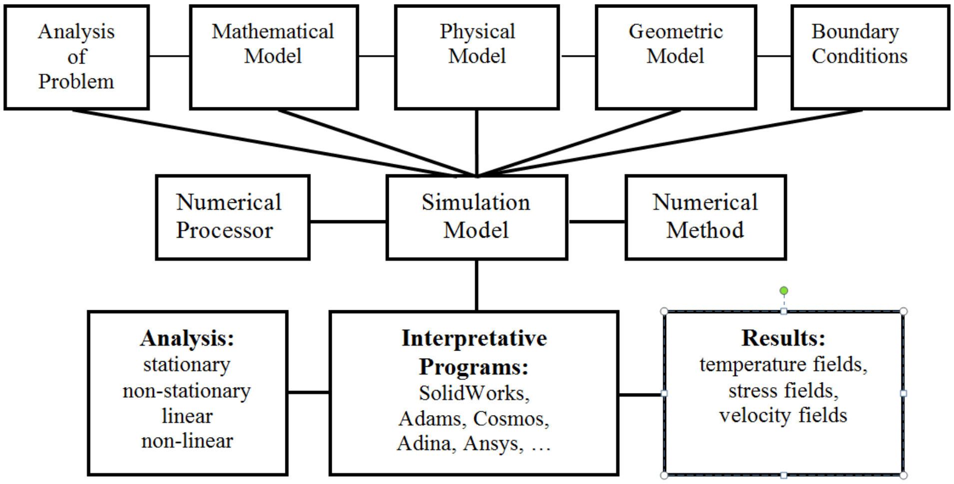


Fig. 1.1 Computational modeling scheme

The principles of computational modeling with subsequent simulation and load analysis contain a large amount of information from mathematics [2, 6, 7, 9, 11, 42], mechanics [10-15, 17, 19, 22- 50], finite element method (FEM) [1, 3, 4, 5, 8, 16, 18, 20, 51], as well as from the theory of construction and design of machine nodes and at last but not least, from the field of material engineering. This multidisciplinary conception naturally leads to teamwork or to complex comprehensive degree of knowledge in the specific engineering fields.

1.1 Numerical Methods in Continuum Mechanics

The exact solution of the state equations of elasticity (resilience) and strength is very complicated and therefore, the approximate numerical methods have been developed. The approach to the solving of a mathematical model of a task or issue can be based on two basic methods:

a) the approximate solution of state equations,

b) the application of extremalisation principles (energy approaches).

In relation to the first mentioned method, it is important to point out that we solve the state equations, which are mostly in the form of differential equations and the given solution is based on the finite difference method in various modifications. Using the known differential relations, we can obtain an approximate solution for discrete points to which the investigated object was divided.

In the second mentioned method, we use the basic principles of mechanics related to the energy balance. For example, we compare the potential energy of internal and external forces by minimizing the difference. The given comparison is performed as a variation task (issue) or through the principle of virtual work or performance. A more detailed description of the introduced methods as well as basic principles can be seen in [3], [7-9] and many other literature sources.

Based on the principle of virtual work, the best-known numerical methods include also the Galerkin-Bubnov method in its deformation or force form.

The methods, which are based on conventional variation principles, include the Rayleigh-Ritz method in two variants – deformation and force. The finite element method is also based on the given deformation variant. The main idea of all methods relates to the estimation – the approximation of the solution by help of simple mathematical functions, while the following extremalisation condition must be met:

$$\pi = \int_V (\sigma_{ij} \cdot \delta\varepsilon_{ij} - X_i \cdot \delta u_i) \cdot dV - \int_S p_i \cdot \delta u_i \cdot dS \rightarrow \min . \quad i, j = x, y, z , \tag{1.1}$$

or in matrix form:

$$\pi = \int_V (\boldsymbol{\sigma}^T \cdot \delta\boldsymbol{\varepsilon} - \mathbf{X}^T \cdot \delta\mathbf{u}) \cdot dV - \int_S \mathbf{p}^T \cdot \delta\mathbf{u} \cdot dS \rightarrow \min . \quad , \tag{1.2}$$

where σ_{ij} are the stress tensor components, ε_{ij} are the strain tensor components, p_i are the components of intensity vector for the external force, and u_i are the displacement vector components. Equation (1.1) is supplemented by other members or items, including the fulfilment of boundary conditions, compatibility between elements, or any other conditions.

The best known and most popular numerical methods of engineering mechanics are:

- **Finite-difference method** (in the investigated area, we define points – nodes, which are fictitiously connected and thus we are able to create a mesh, which has to be dense enough to achieve the sufficient accuracy of the discrete values in nodes for the investigated function and nowadays, the given method is used in a minimum way);
- **Transfer Matrix Method** (it is based on the balancing of state quantities between predefined solid body sections, while the most notable application of this method was introduced for static and spectral analysis of simply supported beams with added discrete values and springs – the application of this method is more efficient than application of FEM, but it is limited by complexity of task or issue);
- **Finite Element Method** - FEM (it is based on Rayleigh-Ritz deformation method where the element mesh is defined for the whole body and system equations are created by special globalisation procedure, which includes creation of system parameters of individual elements, such as matrix of stiffness or rigidity, mass or weight and many others – it is currently the most widely used method with great commercial success and a wide range of application);
- **Boundary element method** (the mesh of the elements is created only at the boundary of the investigated object, while the investigated quantities inside the body are calculated by help of the known exact solution of the problem, and the advantage as well as efficiency of the method is based on the reduction of the task/issue extent by one grade – although there are several software packages resulting from principles of this method, it has not been as popular as FEM in engineering practice).

2 Introduction to the Finite Element Method (FEM)

Nowadays, the Finite Element Method (FEM) is fully algorithmised and used in a large number of commercial computational programs in order to solve very complicated tasks or issues. The great advantage of this method is that the created model can correspond accurately to the real geometry of the object. Actually, the given mentioned fact is not the general rule for methods, which are based on conventional approaches of the theory of elasticity (resilience) and strength.

The authors of [1], [3], [4], [8], [16], [51] as well as many others have the greatest merit in dissemination of the important facts, principles and information about the finite element method (FEM).

Currently, the FEM stands for a specialised scientific field and it includes the following parts:

1) the theoretical part, where different FEM formulations and different relations are for different types of elements are derived or differentiated,
2) the mathematical part, which includes such problems as the existence of the solution and its convergence, error estimation of the solution and application appropriate or suitable numerical algorithms,
3) the computer part, where the special computer programs or software are created and implemented, including Fortran, C ++, MATLAB and so on,
4) the application part, where the user operates with FEM programs to solve the specific problems.

The main principle of the method can be characterised very briefly: the planar, face (flat) or spatial construction is divided into elements of any shape, while the given elements are commonly called the finite elements (Fig. 2.1).

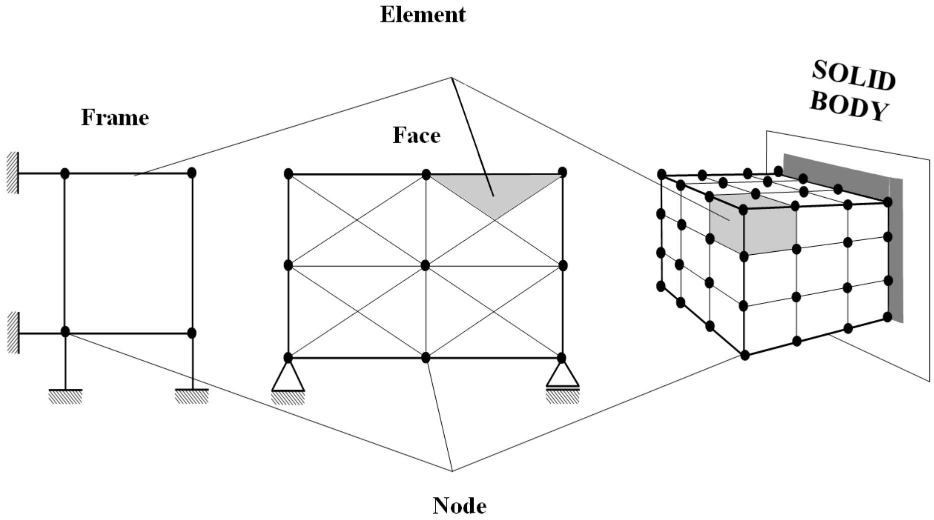


Fig. 2.1 Division of the construction into the finite elements

Dividing lines and areas create dividing points or in other words “nodes”. The values of displacements or forces at nodal points are considered as unknown quantities and they are designated as the nodal parameters. The deformations and stresses within the elements are expressed by using the nodal parameters in the form of polynomials or interpolation functions. Finally, the variational principle is applied to the continuum as a whole, and by this way, it is possible to specify the equations for the searched quantities. After the solution of the given equations, the deformation and stress of the whole construction (structure) is determined.

2.1 Linear Analysis of Static Tasks

There are the basic ideas of the finite element method for static loading of the construction (structure). The deformation variant of FEM is going to be applied and therefore the displacements for individual points or nodes of the discretised system will be analysed as the first. This system is divided into a finite number of elements so that the changes in cross-sections, material properties or forces can be taken into account.

For more complicated tasks (issues), there is possible to use the energy approaches, especially conventional variational approaches, including the

Lagrangian's minimum energy potential approach and the Castigliano's principle for minimum energy of system. We consider the deformation approach and therefore, it is needed to explain Rayleigh-Ritz strain or deformation method briefly. By applying the matrix notation of equation (1.2), let us define the problem relating to the minimum functional of the total potential energy

$$\pi = \frac{1}{2}\int_V \left(\boldsymbol{\sigma}^T \cdot \boldsymbol{\varepsilon}\right) \cdot dV - \int_S \mathbf{u}^{\mathbf{T}} \cdot \mathbf{p} \cdot dS \rightarrow \min., \tag{2.1}$$

where $\boldsymbol{\sigma}$ is the stress vector, $\boldsymbol{\varepsilon}$ is the deformation vector, $\mathbf{p}$ is the external load intensity vector and $\mathbf{u}$ represents the displacement vector. The first integral is the work of the internal forces and the second one represents the work of the external forces (volume forces are not taken into account). Let us take into account the validity of Hooke's law in general:

$$\boldsymbol{\sigma} = \mathbf{D} \cdot \boldsymbol{\varepsilon}, \tag{2.2}$$

where $\mathbf{D}$ is a matrix of material constants, and then (2.1) is in the form:

$$\pi = \frac{1}{2}\int_V \left(\boldsymbol{\varepsilon}^T \cdot \mathbf{D} \cdot \boldsymbol{\varepsilon}\right) \cdot dV - \int_S \mathbf{u}^{\mathbf{T}} \cdot \mathbf{p} \cdot dS \rightarrow \min. \tag{2.3}$$

One of the basic ideas of the finite element method (FEM) is based on the approximation of the function for *u*, *v*, *w* displacement by approximate shape functions, based on discrete values for displacements in the finite element nodes. The finite element stands for the part of the solid body and the given part can be defined by nodal points, own geometry, material characteristics and equations, which are valid in the mechanics for flexible bodies (but not for all, as it is evidenced by some studies or publications [6], [15-16]). The finite element model of the solid body is then created by the imaginary connection of all elements. In fact, this imaginary connection is carried out by creating of so-called global stiffness (rigidity) matrix of specific system and it represents the basic system parameter of the method and moreover, its creation is a mathematical expression of the process of discretization of the analysed body.

Using Cauchy's equation, we can get a general relation between the $\boldsymbol{\varepsilon}$ vector of relative strains or deformations and the $\mathbf{u}$ vector of discrete displacement values in the nodes of the element:

$$\boldsymbol{\varepsilon} = \mathbf{B} \cdot \mathbf{u}, \tag{2.4}$$

where $\mathbf{B}$ is a matrix of differentiations of shape functions in accordance with Cauchy's equations. After insertion of (2.4) to (2.3), we can get:

$$\pi = \frac{1}{2}\int_V \left(\mathbf{u}^T \cdot \mathbf{B}^T \cdot \mathbf{D} \cdot \mathbf{B} \cdot \mathbf{u}\right) \cdot dV - \int_S \mathbf{u}^T \cdot \mathbf{p} \cdot dS = \frac{1}{2} \cdot \mathbf{u}^\mathbf{T} \cdot \mathbf{K} \cdot \mathbf{u} - \mathbf{u}^T \cdot \mathbf{f}_v \rightarrow \min, \tag{2.5}$$

where

$$\mathbf{K} = \int_V \left(\mathbf{B}^T \cdot \mathbf{D} \cdot \mathbf{B}\right) \cdot dV, \tag{2.6}$$

is *the finite element stiffness (rigidity) matrix*. It is different for each one element. The $\mathbf{f}_v$ vector is a nodal force vector. By applying the necessary condition of the existence of the extreme in (2.5), we can obtain

$$\frac{\partial \pi}{\partial \mathbf{u}} = 0 \qquad \Rightarrow \qquad \mathbf{K} \cdot \mathbf{u} - \mathbf{f}_v = \mathbf{0} \quad \text{alebo} \quad \mathbf{K} \cdot \mathbf{u} = \mathbf{f}_v. \tag{2.7}$$

Equation (2.7) represents the equilibrium of forces in nodes of the element. After the so-called globalization procedure (calculation of the stiffness contributions of all elements as well as creation of the overall stiffness matrix of the investigated system), the vector of the nodal forces will be the same as the vector of external forces introduced into the nodes and matrix $\mathbf{K}$ will already be the overall stiffness (rigidity) matrix of system in the global coordinate system. Then, the state equation for the problem can be:

$$\mathbf{K}_{GLOB} \cdot \mathbf{u}_{GLOB} = \mathbf{f}_{GLOB}. \tag{2.8}$$

By solving of the $\mathbf{u}_{GLOB}$ and its subsequent conversion to the $\mathbf{u}$ vectors of displacements for the individual elements, we can calculate the $\boldsymbol{\varepsilon}$ vector – equation (2.4) for each finite element and using the Hooke's stress-strain law, we can get $\boldsymbol{\sigma}$.

The principle of creation of the stiffness matrix of the element as well as the creation of the whole analysed system represents the first step relating to successful understanding and familiarising of the method. The finite element represents a fictitiously selected small part of the solved construction (structure), for which the selected conditions (describing the observed physical phenomenon) have to be valid. The each one element is characterised by material and geometric properties and it is important to point out that its physical interpretation is mainly based on mathematical means.

The finite elements are divided according to their geometry (Fig. 2.2) into:

- the one-dimensional (bars, beams),
- the two-dimensional (plane stress or strain, triangular, quadrangular elements),
- the three-dimensional (spatial stress and deformation, tetrahedron, brick elements).

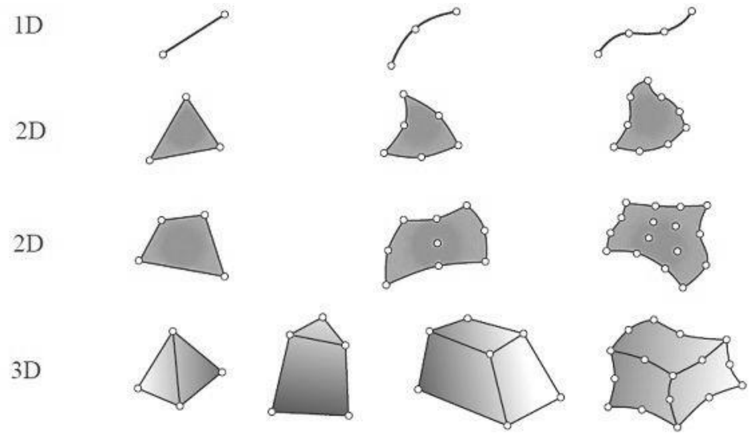


Fig. 2.2 Division of the finite elements [3]

The finite element method makes it possible to detect the stress and state of deformation or strain under the any load at any point in the solid body of any shape and material.

2.2 Modeling with Four-node Tetrahedral Space Elements

Consider the four-node linear tetrahedral spatial element (Fig. 2.3). Element belongs among the basic types of volume finite elements [1], [3], [51].

In the element, we are able to define 12 strain or deformation degrees of freedom – after the three displacements for each node, the vector of unknown nodal displacements can be:

$$\mathbf{u} = \begin{bmatrix} u_1 & v_1 & w_1 & u_2 & v_2 & w_2 & u_3 & v_3 & w_3 & u_4 & v_4 & w_4 \end{bmatrix}^T . \tag{2.9}$$

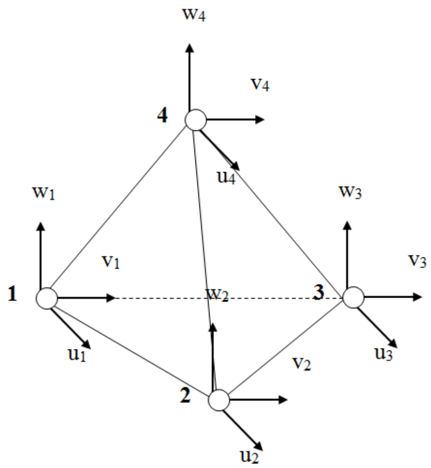


Fig. 2.3 Tetrahedron spatial element

In the order to differentiate the stiffness matrix of the element, we consider the three linear functions displacements relating to *u (x,y,z)*, *v (x,y,z)* and *w (x,y,z)*:

$$u(x, y) = \begin{bmatrix} 1 & x & y & z \end{bmatrix} \cdot \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix} = \begin{bmatrix} 1 & x & y & z \end{bmatrix} \cdot \boldsymbol{\alpha}_+$$

$$v(x, y) = \begin{bmatrix} 1 & x & y & z \end{bmatrix} \cdot \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{Bmatrix} = \begin{bmatrix} 1 & x & y & z \end{bmatrix} \cdot \boldsymbol{\alpha}_2 \tag{2.10}$$

$$w(x, y) = \begin{bmatrix} 1 & x & y & z \end{bmatrix} \cdot \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix} = \begin{bmatrix} 1 & x & y & z \end{bmatrix} \cdot \boldsymbol{\alpha}_3$$

or

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{bmatrix} 1 & x & y & z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & x & y & z & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x & y & z \end{bmatrix} . \begin{Bmatrix} \boldsymbol{\alpha}_1 \\ \boldsymbol{\alpha}_2 \\ \boldsymbol{\alpha}_3 \end{Bmatrix} = \mathbf{a} \cdot \boldsymbol{\alpha} \tag{2.11}$$

We can determine the unknown coefficients of the $\boldsymbol{\alpha}$ vector from the boundary conditions for the nodal values of displacements and it can be seen in the matrix form in (2.12).

$$\begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ w_2 \\ u_3 \\ v_3 \\ w_3 \\ u_4 \\ v_4 \\ w_4 \end{Bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 & z_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & x_1 & y_1 & z_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & x_2 & y_2 & z_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & x_3 & y_3 & z_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x_3 & y_3 & z_3 \\ 1 & x_4 & y_4 & z_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & x_4 & y_4 & z_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x_4 & y_4 & z_4 \end{bmatrix} . \boldsymbol{\alpha} = \mathbf{A} \cdot \boldsymbol{\alpha}. \tag{2.12}$$

then, the $\boldsymbol{\alpha}$ vector will be:

$$\boldsymbol{\alpha} = \mathbf{A}^{-1} \cdot \mathbf{u}. \tag{2.13}$$

If the *u (x,y,z)* , *v (x,y,z)* and *w (x,y,z)* functions of area for displacements *are:*

$$\begin{Bmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{Bmatrix} = \mathbf{a} \cdot \mathbf{A}^{-1} \cdot \mathbf{u}, \tag{2.14}$$

we can express Cauchy’s vector:

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{bmatrix} \partial u/\partial x \\ \partial v/\partial y \\ \partial w/\partial z \\ \partial v/\partial x + \partial u/\partial y \\ \partial v/\partial z + \partial w/\partial y \\ \partial w/\partial x + \partial u/\partial z \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \cdot \mathbf{A}^{-1} \cdot \mathbf{u} = \mathbf{B} \cdot \mathbf{u}. \tag{2.15}$$

If we consider the spatial stress, the matrix for material constants is:

$$\mathbf{D} = \frac{E}{(1-2\nu)\cdot(1+\nu)} \cdot \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}, \tag{2.16}$$

where E is the known tensile modulus and ν is Poisson's ratio. By applying the general relation for the calculation of the stiffness (rigidity) matrix, we can get:

$$\mathbf{K} = \int_V \mathbf{B}^T \cdot \mathbf{D} \cdot \mathbf{B} \cdot dV = V \cdot \mathbf{B}^T \cdot \mathbf{D} \cdot \mathbf{B}, \tag{2.17}$$

where V is the tetrahedron volume, which can be calculated:

$$V = \frac{1}{6} \det \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{bmatrix}, \tag{2.18}$$

where x_1, x_2, ... y_4 are the coordinates for the nodes of the element. The stress calculation is based on Hooke's law and it is implemented as follows:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \boldsymbol{\sigma} = \mathbf{D} \cdot \boldsymbol{\varepsilon} = \mathbf{D} \cdot \mathbf{B} \cdot \mathbf{u}, \tag{2.19}$$

where

$\boldsymbol{\sigma}$ is the stress vector,

$\mathbf{D}$ is a matrix of material constants,

$\boldsymbol{\varepsilon}$ is the strain vector,

$\mathbf{B}$ is a matrix of differentiations of shape functions according to Cauchy's equations

$\mathbf{U}$ is the displacement vector.

In this case, the stresses are constant throughout the whole element. This results in discontinuity of the calculated stresses throughout the whole analysed body.

3 Kinematic Analysis of Planar Solid Body Systems

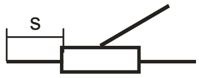
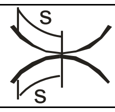
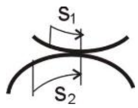
The purpose of the kinematic solution of the solid body systems is to determine the movement (motion) of the individual members or items and their significant points in dependence on the movement (motion) of the members or items which drive the mechanism. To determine movement (motion), it is necessary to determine the position, velocity or speed and acceleration (or angular position, angular velocity or speed, and angular acceleration) of investigated items (members) and points in dependence on the position of driving items (members) or in dependence on the time.

3.1 Basic Concepts and Fundamental Terms

A solid body system is a kind of assembly of at least three bodies (members), including a basic frame, which is used to connect the given items (members) together by the kinematic constraints. Mechanism is a movable system of items or members (bodies), which move mutually, while the kinematic constrains stand for one or two degrees of freedom of movement. Mechanisms, which are used for the transmission of motion (or transmission of forces, moments), are called transmission mechanisms and mechanisms, which are predominantly used to drive the points and bodies on certain paths or trajectories are called driving mechanisms.

The two items (members) that are connected or joined together and can move relative to each other can be understood as a kinematic pair. An overview of some kinematic pairs of planar mechanisms is given in Tab. 3.1 and their design can be various [9].

Tab. 3.1 Planar kinematic pairs

Description of Kinematic Pair	Degrees of Freedom	Type or Class	Symbol	Scheme and Coordinate
Rotary	1	2	r	
Sliding	1	2	p	
Rolling	1	2	v	
General	2	1	o	

The items or members which drive the mechanism are called driving items (members) and the driven items (members) are those, the motion (movement) of which depends on the driving item or member. Connected by kinematic pairs, several bodies (members) form a kinematic chain. The kinematic chain can be opened or closed, and each can be either simple or complex.

The relationship between the output value (for example, the angle or path of the output member) and the input value is called the transmission ratio.

3.2 Classification of Solid Body Systems (Mechanisms)

Mechanisms can be classified on the basis of the various aspects:

- on the basis of the movement (motion) of the bodies (items or members) – the plane, spherical and spatial mechanisms,
- on the basis of number of items (members) – the simple mechanisms (3, 4 items or members) and complex mechanisms (6 or more items or members),
- on the basis of transmission with permanent transmission and variable transmission,
- on the basis of the applied elements – articulated, cam, geared, etc.,

- on the basis of number of degrees of freedom – the mechanisms with one or more degrees of freedom,
- on the basis of any other criteria.

3.3 Formation of Planar Mechanisms

The theory of creation or formation of mechanisms deals with general methods of their structural analysis and synthesis. Mechanisms are **planar** if all items (members) move in planes parallel to each other. In Fig. 3.1 a, b, c, there are kinematic diagrams of the items (members) for binary (the 2nd), ternary (the 3rd) and quaternary (the 4th) degree with rotational kinematic pairs. In Fig. 3.1 d, e, there are binary items (members) with one and two displacement pairs. The kinematic chain consists of a set of items (members) which are connected by kinematic pairs. The kinematic chain is called simple if all the members of this chain are in binary or the 2nd degree (Fig. 3.2 a, b). The kinematic chain is called complex if it contains at least one item (member) of ternary or the 3rd or even higher degree (Fig. 3.2 c, d) [14].

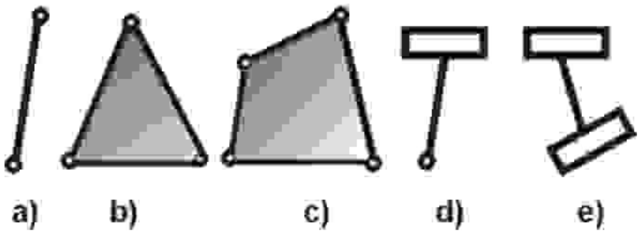


Fig. 3.1 Binary members

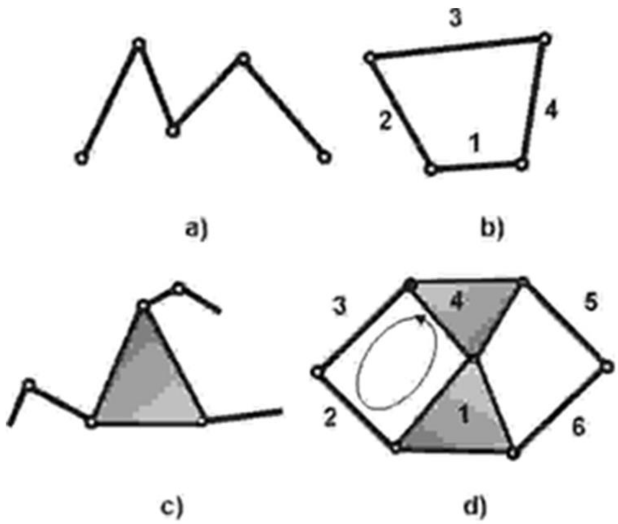


Fig. 3.2 Kinematic chain: simple (a, b) and complex (c, d)

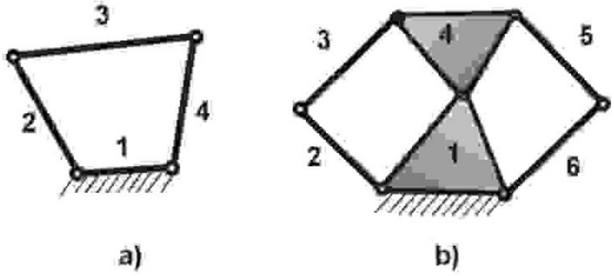


Fig. 3.3 Simple mechanism (a) and complex mechanism (b)

If in the kinematic scheme of a chain several of its items (members) form a closed pattern image (polygon), we can say that the chain forms a loop. Actually, it is the closed cinematic chain (Fig. 3.2 b, d). If the kinematic chain is opened, there is not any loop (Fig. 3.2 a, c). A combined kinematic chain represents the condition where some members are in the loops and some items (members) are not in the loops. Mechanisms arise from closed kinematic chains if anyone of items (members) becomes a frame.

The simple kinematic chains result in **simple mechanisms** (Fig. 3.3a) and the complex chains stand for the arising of **complex mechanisms** (Fig. 3.3b). The introduced method for formation of the mechanisms is called **kinematic chain**

method. In the systematic creation of mechanisms by this method, the large number of them can be assembled, because it is possible to select different chain items (members) to be the frame. Moreover, it is possible to replace the rotating pairs by any other parts or to change the driving items (members) as well as proportions of items (members) in relation to the mechanism. The mechanisms can be also formed or created by the **method for grouping of systems (objects)**. When connected with free kinematic pairs to a frame, the group of systems is a kinematic chain which gives a fixed, static, specific system. Various groups of systems are shown in Fig. 3.4.

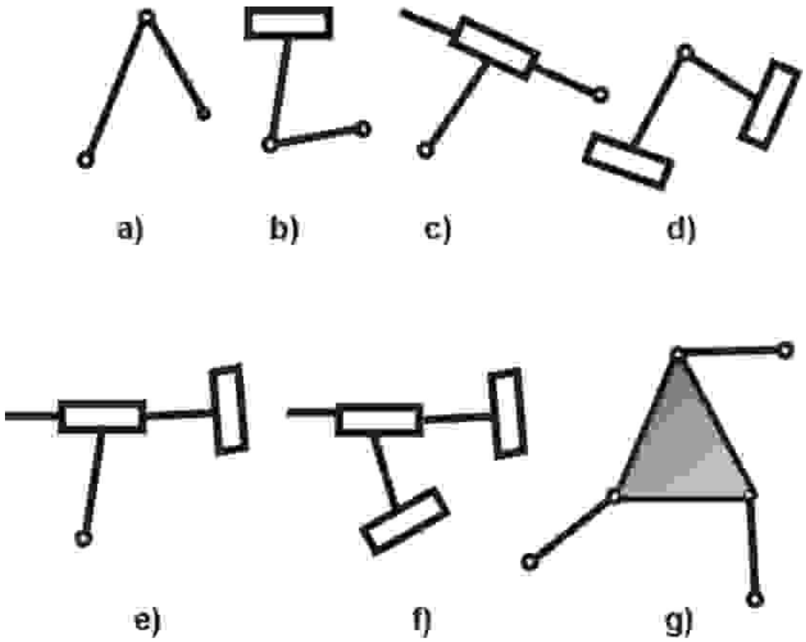


Fig. 3.4 Different groups of systems

The connected or disconnected with the body systems (mechanism), the group of systems does not change the number of degrees of freedom (movability). The given statement represents the prism of the method for grouping of systems or grouping of objects. The gradual connection of systems or objects with the driving items (members) and with the frame and subsequently with the mechanisms resulting from the previous steps leads to the creation or formation of variety of mechanisms. The other possible steps are shown in the following statements:

- *if the binary member with the pairs of the second type or class is connected with the mechanism, the movability of the given mechanism will be decreased by one degree of freedom and if the given item (member) is disconnected, the movability will be increased by one degree of freedom,*
- *if the binary item (member) with the pairs of the second type or class is replaced by common pair, which have connected the items (members) in mechanism, the movability of the mechanism will not be changed.*

The mechanism, which is shown in 3.5 b, can be formed or created if the binary group (designated as 3, 4) is connected with the crank and frame (designated as 2 and 1, respectively) and subsequently, the other binary group (designated as 5, 6) is connected with mechanism, which was created or formed in the first or previous one step. The resulting mechanism has 1° of degrees of freedom because the crank (designated as 2) has also 1° of degrees of freedom, and the further and subsequently connected objects or groups of objects are not able to change the given movability.

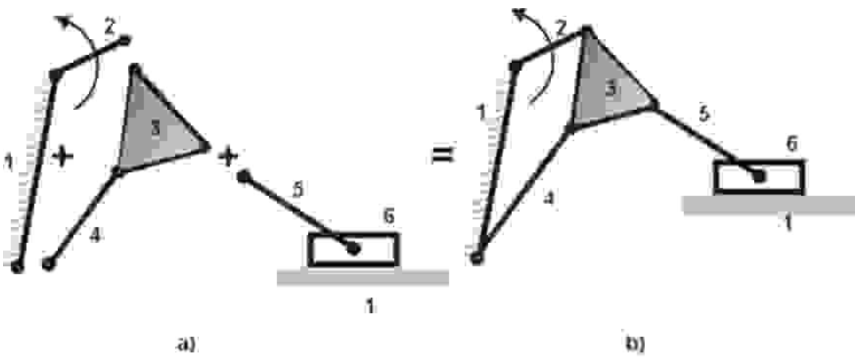
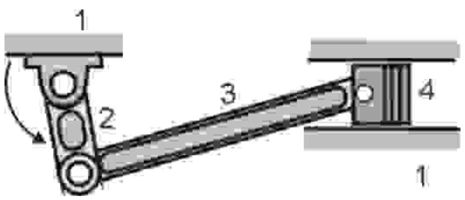
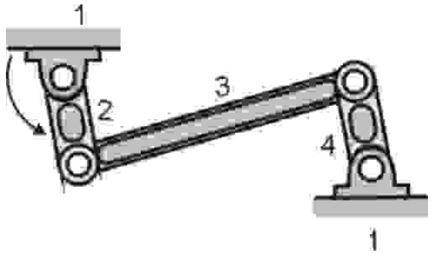


Fig. 3.5 Formation or creation of group of objects

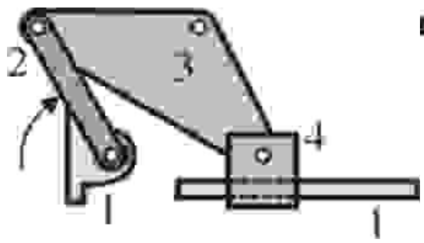
Basic four-item mechanisms or mechanisms with four members:

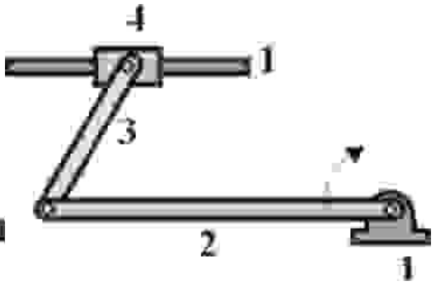




Crank mechanism Four-item articulated mechanism

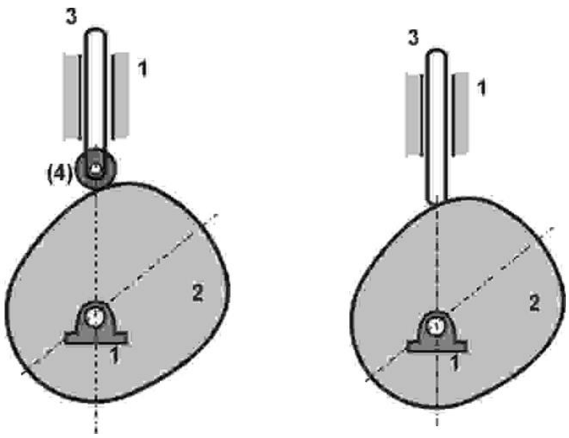
The further possible variants of four-item mechanisms





Basic three-item mechanisms or mechanisms with three members:

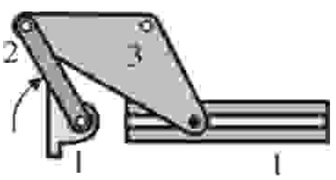
The cam mechanisms are the most common representatives of three-item mechanisms

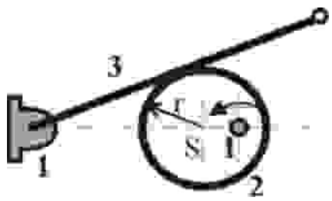


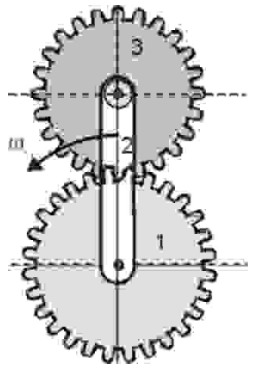
Cam mechanism

Three-item mechanisms with one degree of freedom are needed to have one common kinematic pair.

The further possible variants of three-item mechanisms

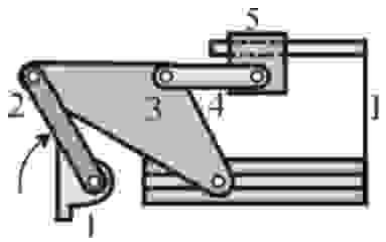


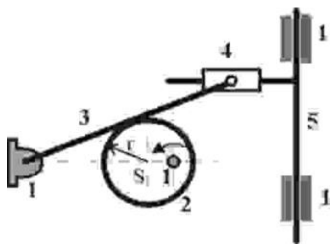


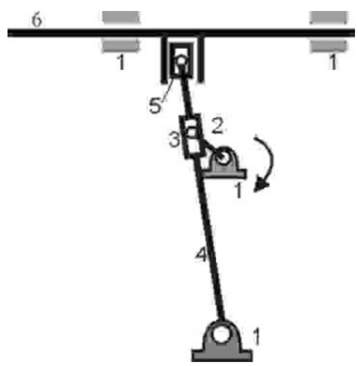


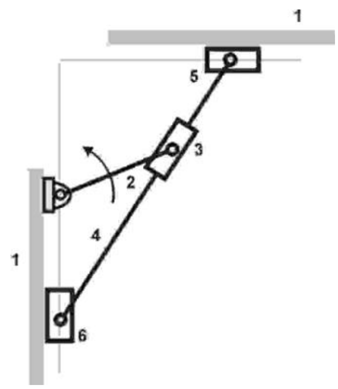
Multi-item mechanisms or mechanisms with more members:

Multi-item mechanisms are made up of the basic four-item and three-item mechanisms by connecting the groups of systems or objects.

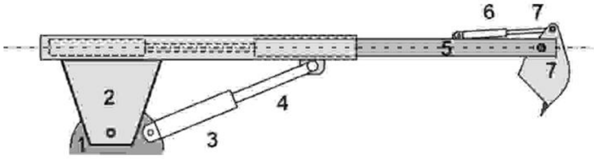


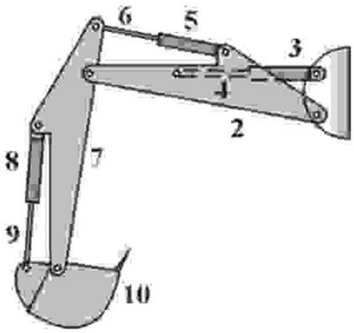






Mechanisms with more degrees of freedom:





3.4 Degree of Freedom of Body System

The number of degrees of freedom of the planar mechanism is calculated according to the equation [9, 10, 14].

$$n = 3\,(\,i - 1\,) - 2d_2 - d_1, \tag{3.1}$$

where

n - the number of degrees of freedom,

3 - the solid body in plane has 3° of freedom,

i - the number of items (members) of mechanism along with the frame,

-1 - the subtraction of the frame,

d_2 - the number of pairs of the second type or class (the total number of rotation, displacement and rolling pairs),

d_1 - the number of pairs of the first type or class (common pairs).

An overview of planar kinematic pairs is in Tab. 1.3. The construction design of these pairs can be different. In the literature [9, 14], there is the equation (3.2), which is equivalent to equation (3.1).

$$n = 3(i-1) - 2(r + p + v) - o \tag{3.2}$$

3.5 Formation of Kinematic equations

The elements of any set of variables are called generalized coordinates but it is also important to point out that the given set of variables strictly determines the position and orientation of all the solid bodies of the mechanism. Generalized **q** coordinates can be independent or dependent. They are changed on the dependence of time and they are going to be written to the column vector:

$$\mathbf{q} = (q_1 \;\; q_2 \; ... \; q_n)^T , \tag{3.3}$$

where n represents their overall number.

The vector equation of its position stands for the basic equation of the mathematical model of the mechanism and it describes its kinematic properties [1].

$$\phi(\mathbf{q},t) = 0 \tag{3.4}$$

After the breaking down of the given equation and specification of its scalar components, we can get a system of nonlinear equations. The equations can be divided into two groups.

1) The equations, which describe the system of solid bodies regardless of the input parameters of the driving item (member), are the basis for the solution of the dependent generalized coordinates. They are called kinematic equations of the position of a system of solid bodies.

$$\phi_K(\mathbf{q},t) = 0 \tag{3.5}$$

2) Equations that describe the values of input generalized independent coordinates are defined by the kinematic equations of the input variables. Their number is equal to the number of degrees of freedom of the n_D mechanism.

$$\phi_D(\mathbf{q},t) = 0 \tag{3.6}$$

We rewrite (3.4) by help of (3.5) and (3.6) in the form:

$$\Phi(\mathbf{q},t)=\begin{bmatrix}\phi_K(\mathbf{q},t)\\ \phi_D(\mathbf{q},t)\end{bmatrix}=0\,. \tag{3.7}$$

The number of dependent variables is equal to the number of equations (3.5) and it is designated as n_K. The total number of variables is: $n=n_D+n_K$.

If the connections between the bodies are holonomic (they do not change with time), the equation (3.7) has the form:

$$\Phi(\mathbf{q},t)=\begin{bmatrix}\phi_K(\mathbf{q})\\ \phi_D(\mathbf{q},t)\end{bmatrix}=0\,. \tag{3.8}$$

The ways of its formation are different. For planar mechanisms, the vector loop method is mostly used. In computational programs for systems of bodies (DADS), there are used the elements to create a model of the mechanism, while the given elements are combined to obtain the specific resulting mechanism. The equations of motion of the mechanism are created by "combination" of the equations for the individual members of the mechanism. The examples of the elements are: connection of a point of body with a frame, connection of points of two bodies, rotary connection of two bodies, displacement connection of two bodies, connection of bodies by means of a rod with articulated end or with two displacement ends and the combination of joint with displacement end, wheels – rolling cams, absolute driving systems, relative driving systems. For the solution of the equation (3.8), the Jacobian's $\Phi(\mathbf{q},t)$ matrix is used:

$$\mathbf{\Phi}_{\mathbf{q}}=\begin{bmatrix}\frac{\partial\varphi_1}{\partial q_1} & \frac{\partial\varphi_1}{\partial q_2} & . & . & . & \frac{\partial\varphi_1}{\partial q_n}\\ \frac{\partial\varphi_2}{\partial q_1} & \frac{\partial\varphi_2}{\partial q_2} & . & . & . & \frac{\partial\varphi_2}{\partial q_n}\\ . & . & & & & \\ . & . & & & & \\ . & . & & & & \\ \frac{\partial\varphi_n}{\partial q_1} & \frac{\partial\varphi_n}{\partial q_2} & . & . & . & \frac{\partial\varphi_n}{\partial q_n}\end{bmatrix}=0\,. \tag{3.9}$$

Jacobian's matrix can be also obtained indirectly by differentiation of equation of the position (3.4).

$$\frac{d}{dt}\mathbf{\Phi}(\mathbf{q},t) = \mathbf{\Phi}_{\mathbf{q}}(\mathbf{q},t)\dot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{t}}\,. \tag{3.10}$$

$\dot{\mathbf{\Phi}}_{\mathbf{t}}$ is partial differentiation of the position equation with dependence of time. The equation of speed (velocity) is obtained by a simple modification:

$$\mathbf{\Phi}_{\mathbf{q}}(\mathbf{q},t)\dot{\mathbf{q}} = -\mathbf{\Phi}_{\mathbf{t}}\,. \tag{3.11}$$

By solving the velocity equation, the velocities or speeds of natural coordinates can be obtained. The obtained results can be used to express the speed (velocity) of any point of the mechanism.

The acceleration equation is obtained by the further differentiating of the (3.11)

$$\mathbf{\Phi}_{\mathbf{q}}(\mathbf{q},t)\ddot{\mathbf{q}} = -(\mathbf{\Phi}_{q}\dot{\mathbf{q}})_{q}\,\dot{\mathbf{q}} - 2\mathbf{\Phi}_{\mathbf{q}t}\dot{\mathbf{q}} - \mathbf{\Phi}_{\mathbf{t}}\,. \tag{3.12}$$

One of the simple ways to solve the equations (3.8) of the position of the mechanism is to use the Newton Raphson method.

$$\mathbf{\Phi}(\mathbf{q}_i) + \mathbf{\Phi}_q(\mathbf{q}_i)(\mathbf{q}_{i+1} - \mathbf{q}_i) = 0 \tag{3.13}$$

Note:

The numerical complications can occur when the equations of the position of a given task is being solved numerically. It is not enough to use only the Newton Raphson method to solve them, but it is necessary to identify and to eliminate excessing connections. There are often problems due to the fact that the position equations are not unambiguous.

4 Matrix expression of Kinematic Variables

4.1 Matrix Expression of Kinematic Variables at Rotary Motion

The axis of the rotation passes through the origin of the coordinate system (Fig.4.1) [11, 14].

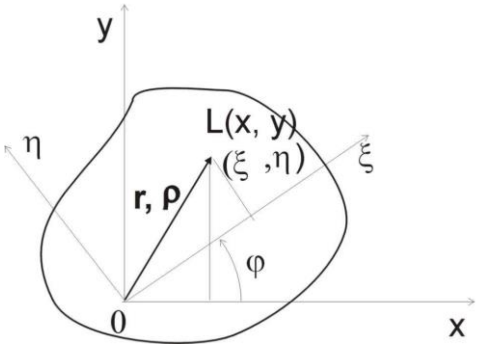


Fig. 4.1

The expression of the position vector for the L point is:

$$\mathbf{r} = \mathbf{T}\boldsymbol{\rho}\,, \tag{4.1}$$

where

$\mathbf{r} = [x,\, y]^T$ *is the position vector for the L point in the basic space,*
$\boldsymbol{\rho} = [\xi,\, \eta]^T$ *is the position vector for the L point in the space of the body.*
The transformation matrix of the rotary motion is:

$$\mathbf{T} = \begin{bmatrix} cos\varphi & -\sin\varphi \\ \sin\varphi & cos\varphi \end{bmatrix}. \tag{4.2}$$

The expression of the speed for L point is:

$$\mathbf{v} = \dot{\mathbf{T}}\boldsymbol{\rho} = \mathbf{T}\boldsymbol{\omega}\rho, \tag{4.3}$$

where

$$\mathbf{v} = [v_x, v_y]^T a \;\; \boldsymbol{\omega} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix}. \tag{4.4}$$

The expression of the acceleration for L point is:

$$\mathbf{a} = \ddot{\mathbf{T}}\boldsymbol{\rho} = \mathbf{T}\left(\boldsymbol{\alpha} + \boldsymbol{\omega}^2\right)\boldsymbol{\rho}\,, \tag{4.5}$$

where

$\mathbf{a} = \left[a_x, a_y\right]^T$ is the expression of the acceleration in the basic space,

$\boldsymbol{\alpha} = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$ is the matrix of the angular acceleration of the solid body. (4.6)

4.2 Matrix Expression of Kinematic Variables at General Planar Motion

For the space of the solid body, the coordinate system is determined as Ω, ξ, η and for basic space, it is O, x, y (Fig. 4.2) [11, 14].

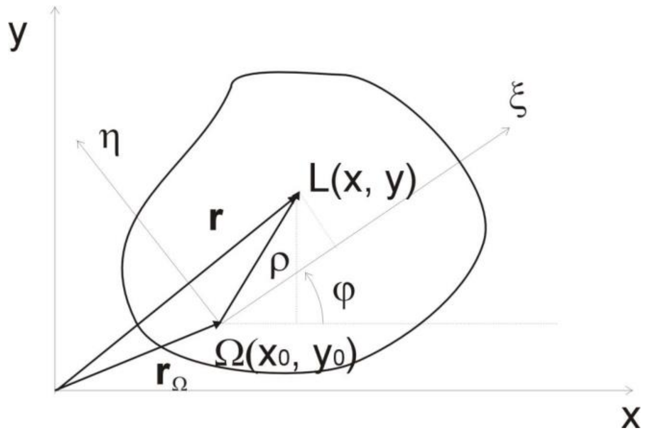


Fig. 4.2

The expression of the position vector for the L point is:

$$\mathbf{r} = \mathbf{T}\boldsymbol{\rho} + \mathbf{r}_{\boldsymbol{\Omega}}\,, \tag{4.7}$$

where

$\mathbf{r} = \left[x, y\right]^T$ is the position vector for the L point in the basic space,

$\boldsymbol{\rho} = \left[\xi, \eta\right]^T$ is the position vector for the L point in the space of solid body,

$\mathbf{r}_{\boldsymbol{\Omega}} = \left[x_0, y_0\right]^T$ is the position vector for reference Ω point in the basic space.

The transformation matrix of the rotary motion is:

$$\mathbf{T} = \begin{bmatrix} cos\varphi & -\sin\varphi \\ \sin\varphi & cos\varphi \end{bmatrix}. \tag{4.8}$$

The expression of the speed (velocity) for the L point in general planar motion is:

$$\mathbf{v} = \dot{\mathbf{T}}\boldsymbol{\rho} + \dot{\mathbf{r}}_{\Omega} = \mathbf{T}\boldsymbol{\omega}\rho + \mathbf{v}_{\Omega} \tag{4.9}$$

The expression of the acceleration for the L point in general planar motion is:

$$\mathbf{a} = \ddot{\mathbf{T}}\boldsymbol{\rho} + \ddot{\mathbf{r}}_{\Omega} = \mathbf{T}\left(\boldsymbol{\alpha} + \boldsymbol{\omega}^{\mathbf{2}}\right)\boldsymbol{\rho} + \mathbf{a}_{\Omega}\,. \tag{4.10}$$

4.3 Matrix expression of Kinematic Variables at Simultaneous Motions

The body designed as **2** moves within the basic space **1** as well as it moves in relation to body designated as **3** etc., up to its motion in relation to the body designated as **n-1** body, while the body designated as **n** is the body which is investigated. The selection of the coordinate systems is $O_i,\, x_i, y_i,\, z_i, \quad i = 1, 2, \ldots\ldots, n$ (Fig. 4.3) [11, 14].

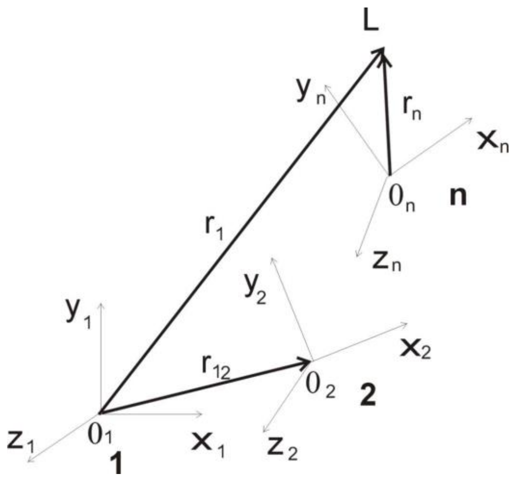


Fig. 4.3

The path of the L point of the n body is expressed by a matrix equation:

$$\mathbf{r_1} = \mathbf{T_{1n}r_n}\,, \tag{4.11}$$

where

$$\mathbf{T}_{1n} = \mathbf{T}_{12}\mathbf{T}_{23}\mathbf{T}_{34}......\mathbf{T}_{n-1,n} = \sum_{i=1}^{n} \mathbf{T}_{i-1,i} \tag{4.12}$$

$\mathbf{r_n}$ or $\mathbf{r_1}$ represent the position vector of point of n body, which is in connection in the system or in the basic space of 1,

$\mathbf{T}_{1n}$ or $\mathbf{T}_{i\text{-}1,i}$ is transformation matrix of n:1motion or i:(i-1) motion.

The speed (velocity) of any point of the n body during simultaneous motions of the individual bodies can be expressed by the equation:

$$\mathbf{v_1} = \dot{\mathbf{T}}_{\mathbf{1n}}\mathbf{r_n} \tag{4.13}$$

or equation:

$$\mathbf{v_1} = \mathbf{T_{1n}}\mathbf{v_{1n}}\mathbf{r_n}\,. \tag{4.14}$$

Matrix of $\mathbf{v_{1n}}$ speed **for** *n:1* can be calculated by help of equations (4.15) or (4.16).

$$\mathbf{v}_{\mathbf{1}n} = \mathbf{T}^{-1}_{\mathbf{1n}}\dot{\mathbf{T}}_{\mathbf{1n}} \tag{4.15}$$

$$\begin{aligned} &\mathbf{v}_{1n} = \mathbf{T}^{-1}_{n-1,n}\mathbf{T}^{-1}_{n-2,n-1,......,}\mathbf{T}^{-1}_{23}\mathbf{v}_{12}\mathbf{T}_{23,....,}\mathbf{T}_{n-2,n-1}\mathbf{T}_{n-1,n} + \\ &+ \mathbf{T}^{-1}_{n-1,n},.....,\mathbf{T}^{-1}_{34}\mathbf{v}_{23}\mathbf{T}_{34,....,}\mathbf{T}_{n-1,n} + \\ &. \\ &. \\ &. \\ &\mathbf{v}_{n-1,n}\,. \end{aligned} \tag{4.16}$$

Matrix of angular speed (velocity) is based on the equation:

$$\omega_{\mathbf{1}n} = \dot{\mathbf{T}}_{1n}\mathbf{T}^{T}_{1n} \tag{4.17}$$

or equation:

$$\omega_{1n} = \mathbf{T}^{T}_{n-1,n}\mathbf{T}^{T}_{n-2,n-1,......,}\mathbf{T}^{T}_{23}\omega_{12}\mathbf{T}_{23,....,}\mathbf{T}_{n-2,n-1}\mathbf{T}_{n-1,n} +$$
$$+\mathbf{T}^{T}_{n-1,n},.....,\mathbf{T}^{T}_{34}\omega_{23}\mathbf{T}_{34,....,}\mathbf{T}_{n-1,n} +$$
$$.$$
$$. \tag{4.18}$$
$$.$$
$$\omega_{n-1,n}.$$

Vectors of angular speed (velocity) can be also expressed by the equation:

$$\omega_{\mathbf{1}n} = \mathbf{T}^{T}_{2n}\omega_{12} + \mathbf{T}^{T}_{3n}\omega_{23} + ...+ \omega_{n-1,n}. \tag{4.19}$$

The acceleration of any point of the n body during simultaneous motions of bodies can be expressed by the equation:

$$\mathbf{a_1} = \ddot{\mathbf{T}}_{\mathbf{1n}}\mathbf{r_n} \tag{4.20}$$

or

$$\mathbf{a_1} = \mathbf{T_{1n}}(\mathbf{A_{1n}} + \mathbf{v}^{\mathbf{2}}_{\mathbf{1n}})\mathbf{r_n}, \tag{4.21}$$

where $\mathbf{A_{1n}} = \dot{\mathbf{v}}_{\mathbf{1n}}$ is matrix of acceleration.

5 Vector Method

The vector method [10], [14] is a general method, which is suitable for kinematic investigation of planar and spatial mechanisms. A certain polygon can be assigned to each one simple mechanism can be assigned, while the sides of the given polygon are considered as the vectors which form a closed pattern (Fig. 5.1). For vectors, constraining the closure conditions, it can be written in the form (5.1):

$$\sum_{i=1}^{n} \mathbf{l_i} = \mathbf{0} \tag{5.1}$$

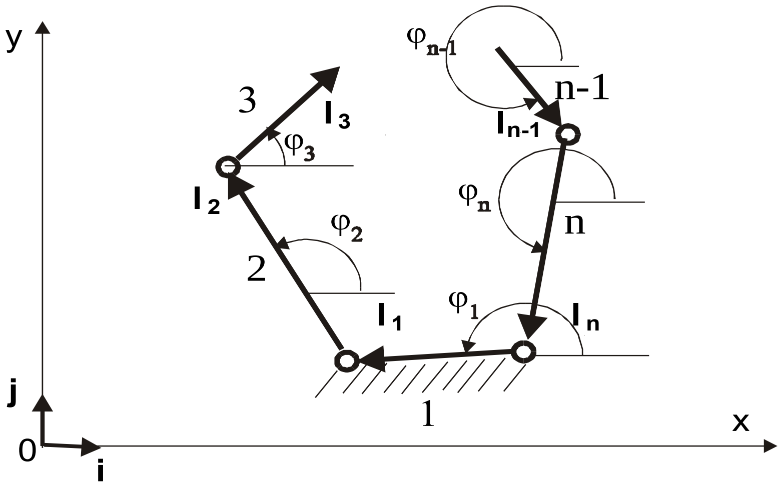


Fig. 5.1

Breaking down the closure equation (5.1) to the *x*, *y* axes, it is possible to get two scalar equations (5.2), which are solution for the position problem (positional analysis of the mechanism),

$$x: \quad \sum_{i=1}^{n} l_i.\cos\varphi_i = 0 \quad \text{and} \quad y: \quad \sum_{i=1}^{n} l_i.\sin\varphi_i = 0 \tag{5.2}$$

where

n – is the number of vectors forming a polygon,

l_i – is a length of the items (members),

φ_i – is the angle between the positive direction of the x-axis and the positive direction of the corresponding vector.

By differentiating equations (5.2), with respect to time, it is possible to get the equations of speed (velocity):

$$\dot{x}: \sum_{i=1}^{n} \dot{l}_i . \cos\varphi_i - \sum_{i=1}^{n} l_i . \dot{\varphi}_i \sin\varphi_i = 0 \quad \text{and} \quad \dot{y}: \quad \sum_{i=1}^{n} \dot{l}_i . \sin\varphi_i + \sum_{i=1}^{n} l_i . \dot{\varphi}_i \cos\varphi_i = 0 \qquad (5.3)$$

If the lengths and angles of some members are constant, i.e.: they are not changed along with time, the corresponding differentiations are zero. By differentiating the equations (5.3), with respect to time, it is possible to get the acceleration equations in the form (5.4):

$$\begin{aligned} \ddot{x}: \sum_{i=1}^{n} \ddot{l}_i . \cos\varphi_i - 2\sum_{i=1}^{n} \dot{l}_i . \dot{\varphi}_i \sin\varphi_i - \sum_{i=1}^{n} l_i \dot{\varphi}_i^2 \cos\varphi_i - \sum_{i=1}^{n} l_i . \ddot{\varphi}_i \sin\varphi_i = 0 \\ \ddot{y}: \sum_{i=1}^{n} \ddot{l}_i . \sin\varphi_i + 2\sum_{i=1}^{n} \dot{l}_i . \dot{\varphi}_i \cos\varphi_i - \sum_{i=1}^{n} l_i \dot{\varphi}_i^2 \sin\varphi_i + \sum_{i=1}^{n} l_i . \ddot{\varphi}_i \cos\varphi_i = 0 \end{aligned} \qquad (5.4)$$

5.1 Kinematic Analysis for Four-Item Mechanism

Relating to the four-item planar mechanism (mechanism with four members), solve the field of positions, angular speeds (velocities) and angular accelerations of item (member) designated as **3** and item (member) designated as **4** in dependence on the position of the driving item (member). Use the **vector method**. The numerical solution is carried out for the specified input values, if the specified angle of rotational displacement for the driving item (member), designated as **2,** is: $\varphi_2 = q$ = 252°. The kinematic scheme of the mechanism is shown in Fig. 5.2.

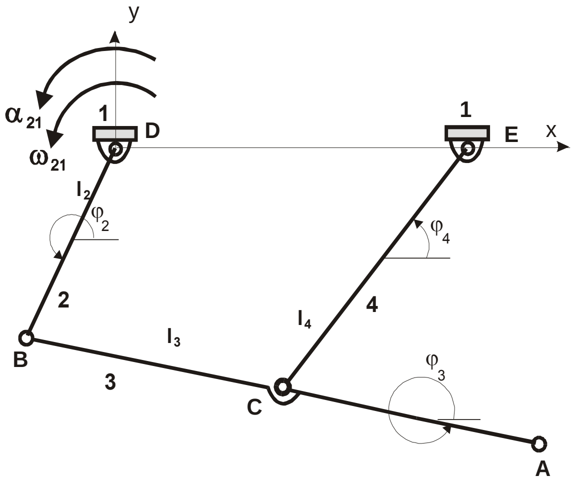


Fig. 5.2

Specified or given values are:

DB =l_2= 0.04 m, BC =l_3= 0.05 m, BA = 0.05 m, CE =l_4= 0.06 m,

$\varphi_2 = q$ =252°, $\varphi_3 = 6.0563$ rad, $\varphi_4 = 0.96\ rad$, $\omega_{21} = 1\ rad.s^{-1}, \alpha_{21} = 1\ rad.s^{-2}$.

Solution:

The mechanism consists of four bodies (items), including a frame. The number of degrees of freedom is calculated according to equation (3.1) or (3.2) and there is 1 ° of freedom. The kinematics of the mechanism can be solved by using the vector loop in Fig. 5.3. The formation of the equations of position (5.5), the differentiation of which, with respect to time for $\phi_i\,(i=1{,}2)$ variables, gives the equations for speeds (velocities) (5.6) and then the given equations are rewritten into a matrix form (5.7). If we differentiate equations (5.6) with respect to time once more, we can get equations for accelerations (5.8), which can be rewritten into the matrix form (5.9). The given solution was carried out by using the SolidWorks and the results of the kinematic analysis are shown in Figs. 5.4, 5.5, 5.6 and the animation is shown in Fig. 5.7.

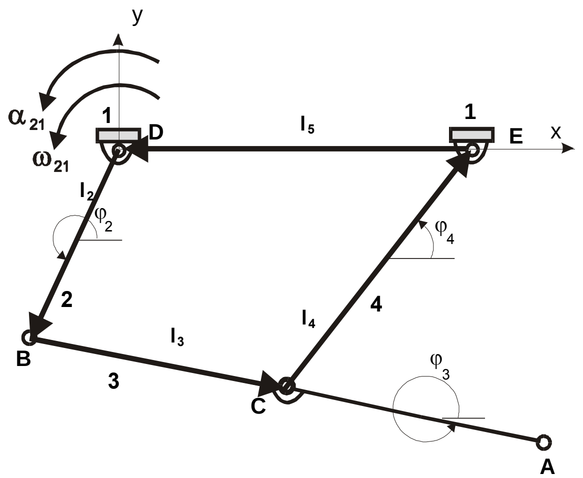


Fig. 5.3

The variables are: $[\phi_3, \phi_4] = [\varphi_3, \varphi_4]$.

For DBCEFD loop, the equations of the position are:

$$\begin{aligned} f_1 &= l_2 \cos q + l_3 \cos \phi_3 + l_4 \cos \phi_4 - l_5 = 0 \\ f_2 &= l_2 \sin q + l_3 \sin \phi_3 + l_4 \sin \phi_4 = 0 \end{aligned} \tag{5.5}$$

Speeds (velocities) and after further differentiation, a set of equations for accelerations is:

$$\begin{aligned} \dot{f}_1 &= -l_2 . \dot{q} \sin q - l_3 . \dot{\phi}_3 \sin \phi_3 - l_4 . \dot{\phi}_4 \sin \phi_4 = 0 \\ \dot{f}_2 &= l_2 . \dot{q} \cos q + l_3 . \dot{\phi}_3 \cos \phi_3 + l_4 . \dot{\phi}_4 \cos \phi_4 = 0 \end{aligned} \tag{5.6}$$

The system of equations for speeds (velocities) in the matrix form is:

$$\begin{bmatrix} -l_3 \sin \phi_3 & -l_4 \sin \phi_4 \\ l_3 \cos \phi_3 & l_4 \cos \phi_4 \end{bmatrix} \cdot \begin{bmatrix} \dot{\phi}_3 \\ \dot{\phi}_4 \end{bmatrix} = \begin{bmatrix} l_2 \sin q \\ -l_2 \cos q \end{bmatrix} \cdot \dot{q} \tag{5.7}$$

$$\ddot{f}_1 = -l_2.\dot{q}^2\cos q - l_2.\ddot{q}\sin q - l_3.\dot{\phi}_3^2\cos\phi_3 - l_3.\ddot{\phi}_3\sin\phi_3 - l_4.\dot{\phi}_4^2\cos\phi_4 - l_4.\ddot{\phi}_4\sin\phi_4 = 0$$
$$\ddot{f}_2 = -l_2.\dot{q}^2\sin q + l_2.\ddot{q}\cos q - l_3.\dot{\phi}_3^2\sin\phi_3 + l_3.\ddot{\phi}_3\cos\phi_3 - l_4.\dot{\phi}_4^2\sin\phi_4 + l_4.\ddot{\phi}_4\cos\phi_4 = 0 \qquad (5.8)$$

The system of equation for acceleration in the matrix form is:

$$\begin{bmatrix} -l_3\sin\phi_3 & -l_4\sin\phi_4 \\ l_3\cos\phi_3 & l_4\cos\phi_4 \end{bmatrix} \cdot \begin{bmatrix} \ddot{\phi}_3 \\ \ddot{\phi}_4 \end{bmatrix} = \begin{bmatrix} l_2.\dot{q}^2\cos q + l_2.\ddot{q}\sin q + l_3.\dot{\phi}_3^2\cos\phi_3 + l_4.\dot{\phi}_4^2\cos\phi_4 \\ l_2.\dot{q}^2\sin q - l_2.\ddot{q}\cos q + l_3.\dot{\phi}_3^2\sin\phi_3 + l_4.\dot{\phi}_4^2\sin\phi_4 \end{bmatrix} \qquad (5.9)$$

The results from the kinematic analysis

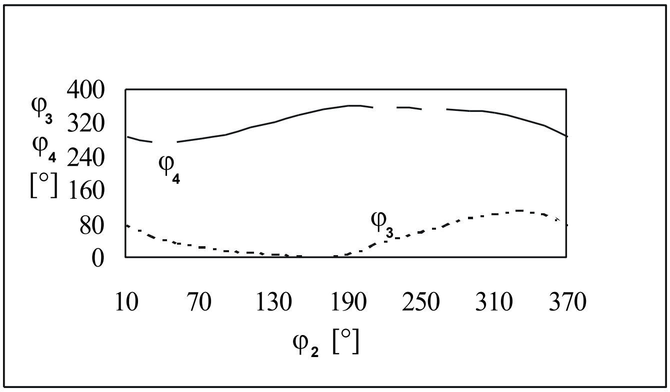


Fig. 5.4

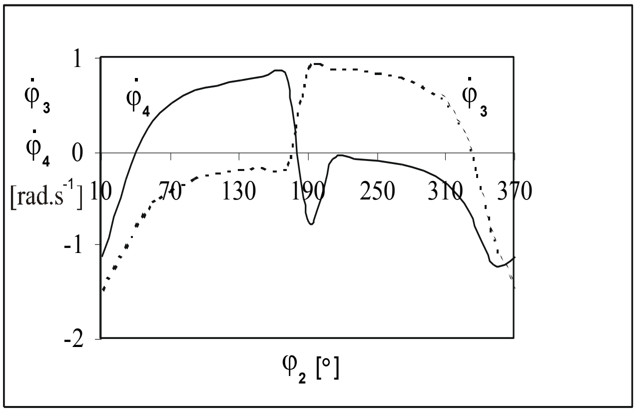


Fig. 5.5

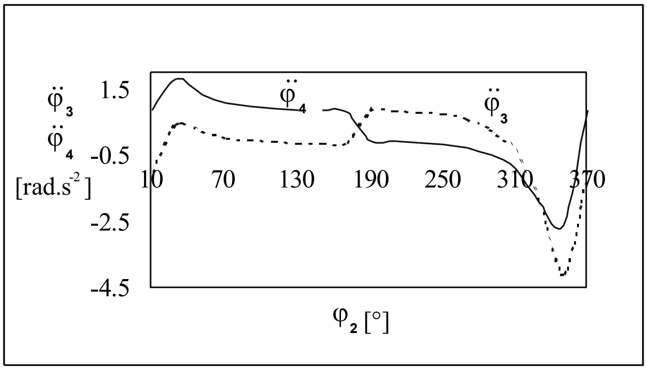


Fig. 5.6

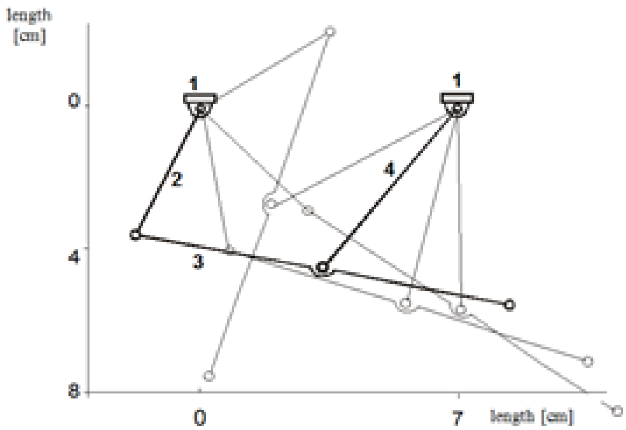


Fig. 5.7 Animation of the four positions of the mechanism

5.2 Kinematic Analysis for Six-Item Mechanism

Relating to the six-item planar mechanism [31] (mechanism with six members), solve the field of positions, speeds (velocities) and accelerations of the driving item (member) in the dependence on the position. Use the **vector method.** The numerical solution is carried out for the specified input values if the specified angle of rotational displacement for the driving item (member), designated as **2,** is: $\varphi_2 = q$ = 20°. The kinematic scheme of the mechanism is shown in Fig. 5.8.

Specified or given values are:

$\phi_1 = r_1 = 0.11\, m, \quad \phi_2 = r_3 = 0.14\, m, \quad \phi_3 = \varphi_4 = 1.2\, rad, \quad \phi_4 = r_5 = 0.085\, m,$

$q = \theta_2 = 0.34908\, rad, \quad a_2 = 0.05\, m, \quad a_4 = 0.3\, m, \quad L = 0.12\, m, \quad \omega_{21} = 1\, rad.s^{-1}, \quad \alpha_{21} = 1\, rad.s^{-2}.$

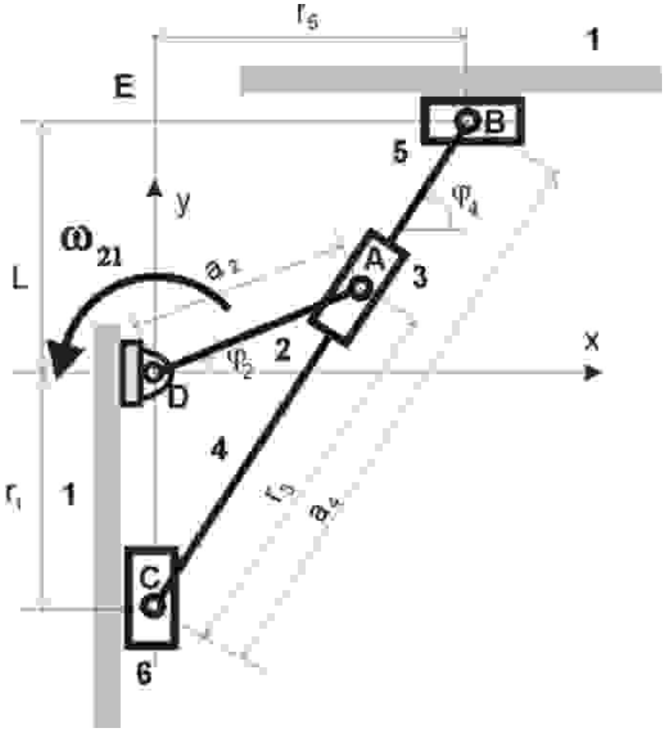


Fig. 5.8

Solution:

The mechanism consists of the six moving bodies (items) and a frame. The number of degrees of freedom is calculated according to equation (3.1) or (3.2) and it has 1 ° of freedom. The kinematics of the mechanism can be solved by the help of the vector loop (Fig. 5.9). Formation of the equations of the position (5.10), (5.11) the differentiation of which for $\phi_i (i = 1,2,3,4)$ variables gives the specific equations for the speeds (velocities) (5.12) and then the given equations are rewritten into a matrix form (5.13).

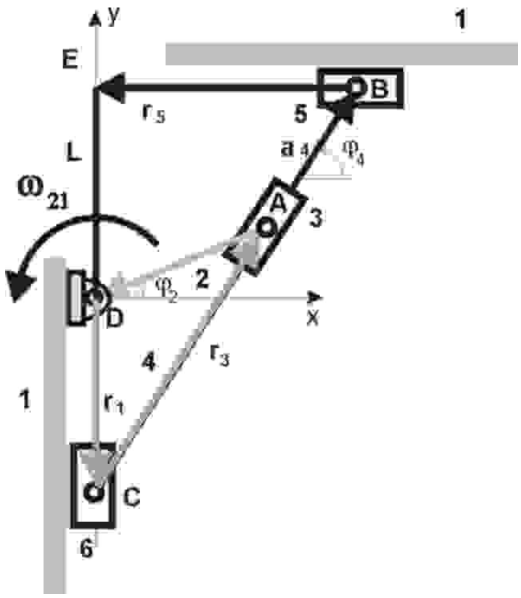


Fig. 5.9

The variables are:

$[\phi_1, \phi_2, \phi_3, \phi_4] = [r_1, r_3, \varphi_4, r_5]$.

The displacement equations for CBEC are:

$$\begin{aligned} f_1 &= a_4.\cos\phi_3 - \phi_4 = 0, \\ f_2 &= a_4.\sin\phi_3 - L - \phi_1 = 0. \end{aligned} \tag{5.10}$$

The displacement equations for the CADC loop are:

$$\begin{aligned} f_3 &= \phi_2.\cos\phi_3 - a_2.\cos q = 0, \\ f_4 &= \phi_2.\sin\phi_3 - a_2.\sin q - \phi_1 = 0. \end{aligned} \tag{5.11}$$

$$\begin{aligned} \dot{f}_1 &= -a_4.\dot{\phi}_3.\sin\phi_3 - \dot{\phi}_4 = 0, \\ \dot{f}_2 &= a_4.\dot{\phi}_3.\cos\phi_3 - \dot{\phi}_1 = 0, \\ \dot{f}_3 &= \dot{\phi}_2.\cos\phi_3 - \phi_2.\dot{\phi}_3.\sin\phi_3 = -a_2.\dot{q}.\sin q, \\ \dot{f}_4 &= \dot{\phi}_2.\sin\phi_3 + \phi_2.\dot{\phi}_3.\cos\phi_3 - \dot{\phi}_1 = a_2.\dot{q}.\cos q. \end{aligned} \tag{5.12}$$

$$\begin{bmatrix} 0 & 0 & -a_4\sin\phi_3 & -1 \\ -1 & 0 & a_4\cos\phi_3 & 0 \\ 0 & \cos\phi_3 & -\phi_2\sin\phi_3 & 0 \\ -1 & \sin\phi_3 & \phi_2\cos\phi_3 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \\ \dot{\phi}_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -a_2\sin q \\ a_2\cos sq \end{bmatrix} \cdot \dot{q} \tag{5.13}$$

If the equations (5.12) are differentiated once more, we can get equations (5.14) for the accelerations and they can be rewritten into a matrix form (5.15).

$$
\begin{aligned}
&\ddot{f}_1 = -a_4.\ddot{\phi}_3.\sin\phi_3 - a_4\dot{\phi}_3^2.\cos\phi_3 - \ddot{\phi}_4 = 0,\\
&\ddot{f}_2 = a_4.\ddot{\phi}_3.\cos\phi_3 - a_4\dot{\phi}_3^2.\sin\phi_3 - \ddot{\phi}_1 = 0,\\
&\ddot{f}_3 = \ddot{\phi}_2.\cos\phi_3 - \dot{\phi}_2.\dot{\phi}_3.\sin\phi_3 - \dot{\phi}_2.\dot{\phi}_3.\sin\phi_3 - \phi_2.\ddot{\phi}_3.\sin\phi_3 -\\
&-\phi_2.\dot{\phi}_3^2.\cos\phi_3 = -a_2.\ddot{q}.\sin q - a_2.\dot{q}^2.\cos q,\\
&\ddot{f}_4 = \ddot{\phi}_2.\sin\phi_3 + \dot{\phi}_2.\dot{\phi}_3.\cos\phi_3 + \dot{\phi}_2.\dot{\phi}_3.\cos\phi_3 + \phi_2.\ddot{\phi}_3.\cos s\phi_3 -\\
&-\phi_2.\dot{\phi}_3^2.\sin\phi_3 - \ddot{\phi}_1 = a_2.\ddot{q}.\cos q - a_2.\dot{q}^2.\sin q.
\end{aligned}
\tag{5.14}
$$

$$
\begin{bmatrix}
0 & 0 & -a_4\sin\phi_3 & -1\\
-1 & 0 & a_4\cos\phi_3 & 0\\
0 & \cos\phi_3 & -\phi_2\sin\phi_3 & 0\\
-1 & \sin\phi_3 & \phi_2\cos\phi_3 & 0
\end{bmatrix}
.
\begin{bmatrix}
\ddot{\phi}_1\\ \ddot{\phi}_2\\ \ddot{\phi}_3\\ \ddot{\phi}_4
\end{bmatrix}
=
\begin{bmatrix}
a_4\dot{\phi}_3^2\cos\phi_3\\
a_4\dot{\phi}_3^2\sin\phi_3\\
2\dot{\phi}_2\dot{\phi}_3\sin\phi_3 + \phi_2\dot{\phi}_3^2\cos\phi_3 - a_2\ddot{q}\sin q - a_2\dot{q}^2\cos q\\
-2\dot{\phi}_2\dot{\phi}_3\cos\phi_3 + \phi_2\dot{\phi}_3^2\sin\phi_3 + a_2\ddot{q}\cos sq - a_2\dot{q}^2\sin q
\end{bmatrix}
\tag{5.15}
$$

The results from the kinematic analysis:

The given solution is carried out by using the SolidWorks and the results of the kinematic analysis are shown in Figs. 5.10, 5.11, 5.12 and the animation of the first three positions is shown in Fig. 5.13.

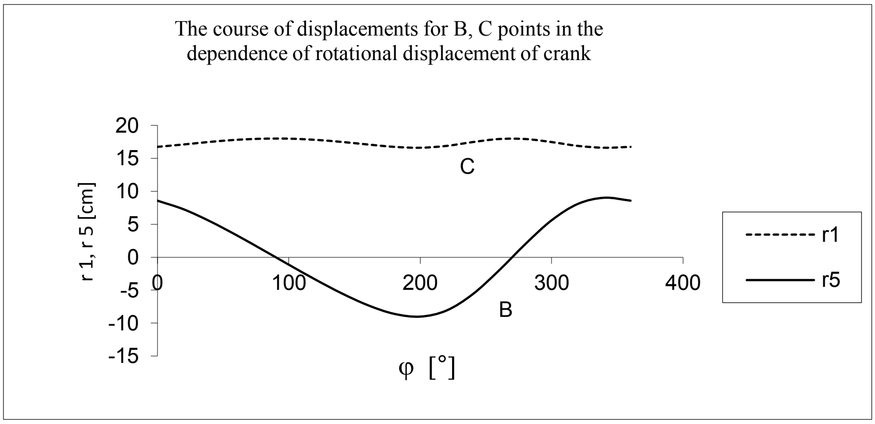


Fig. 5.10

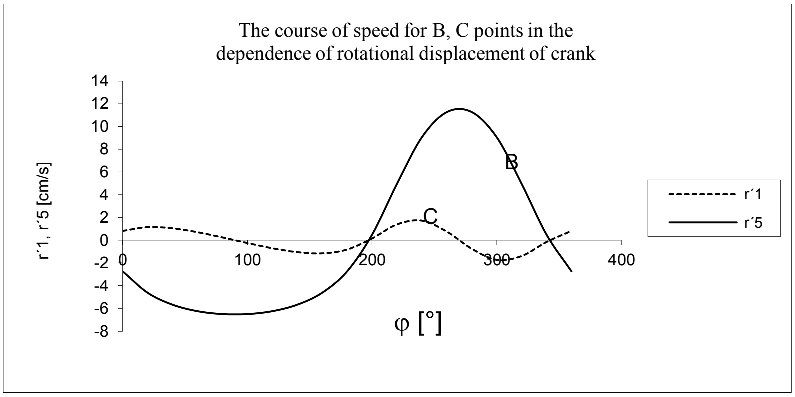


Fig. 5.11

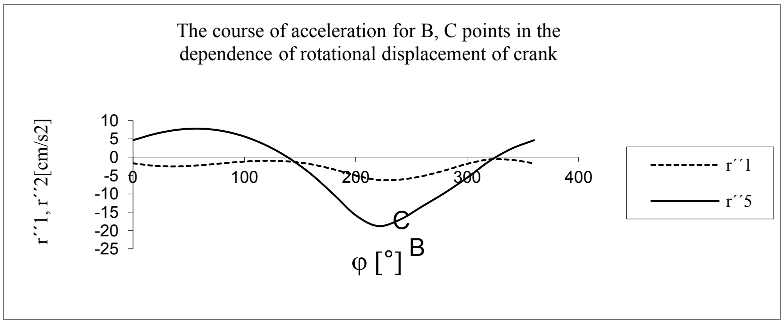
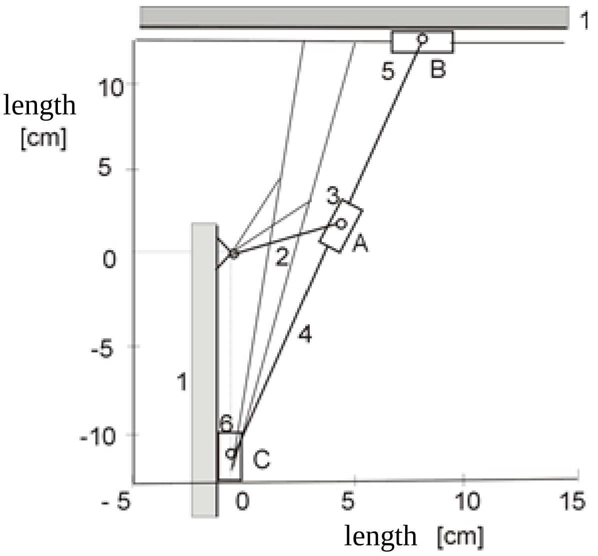


Fig. 5.12



Animation of the three positions of the mechanism

Fig. 5.13

5.3 Kinematic Analysis for Seven-Item Mechanism

Relating to the seven-item planar mechanism [41] (mechanism with seven members), solve the field of positions, speeds (velocities) and in the dependence on the position of the driving item (member). Use the **vector method.** The numerical solution is carried out for the specified input values if the given specified angle of rotational displacement for the driving item (member), designated as **2,** is: $\varphi_2 = q$ = 60°. The kinematic scheme of the mechanism is shown in Fig. 5.14.

Specified or given values are:

$AE = l_1 = 0.06\,m$, $r_2 = r_4 = 0.03\,m$, $l_3 = 0.08\,m, l_5 = 0.06\,m$,
$l_6 = 0.08\,m, l_r = 0.122\,m$, $q = \theta_2 = 60°$, $\varphi_3 = 40^\circ$, $\varphi_5 = 300^\circ$, $\varphi_6 = 20^\circ$,
$\omega_{21} = 2\,rad.s^{-1} = konst$.

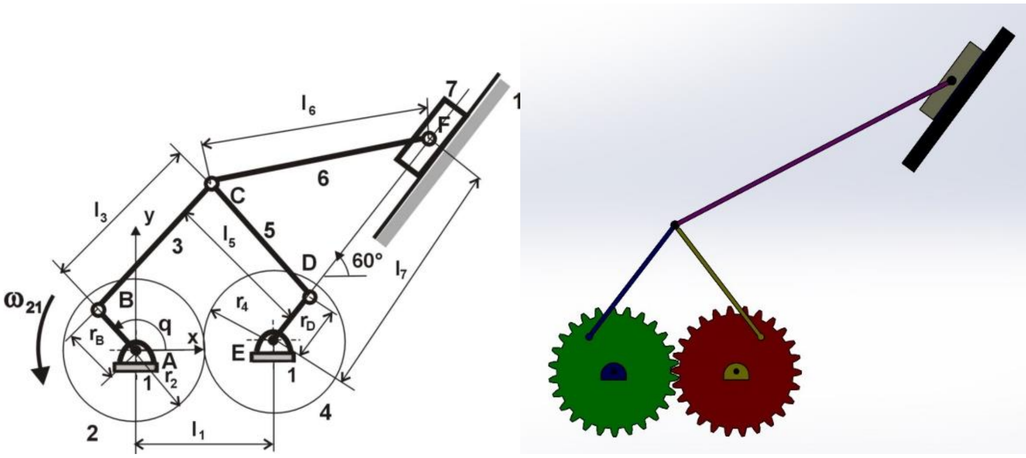


Fig. 5.14. Planar mechanism – computational model.

Solution:

The mechanism consists of the seven moving bodies (items) and a frame. The number of degrees of freedom is calculated according to equation (3.1) or (3.2) and it has 1 ° of freedom. The kinematics of the mechanism can be solved by the help of the vector loop (Fig. 5.15). Formation of the equations of the position (5.16), (5.17) the differentiation of which, for $\phi_i(i = 1,2,3,4)$ variables, gives the specific equations for the speeds or velocities (5.18) and then the given equations are rewritten into a matrix form (5.19). If the equations (5.18) are differentiated once more, we can get equations (5.20) for the accelerations and they can be rewritten into a matrix form (5.21).

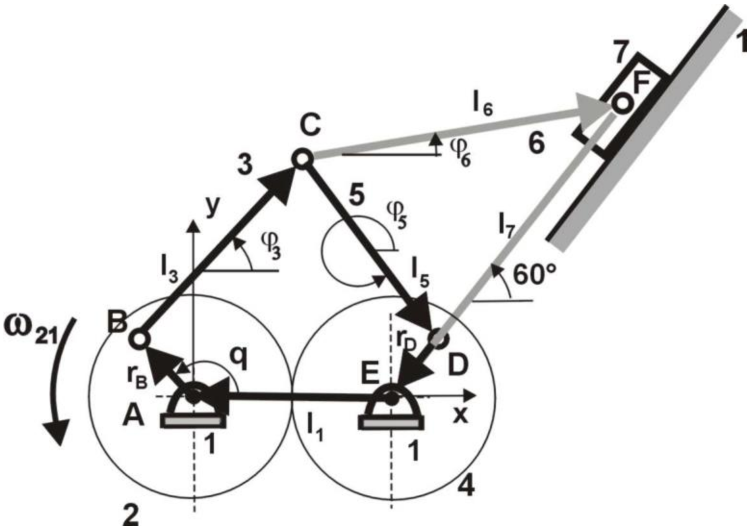


Fig. 5.15

The variables are:

$$[\phi_1, \phi_2, \phi_3, \phi_4] = [\varphi_3, \varphi_5, \varphi_6, L_7]$$

The equations of the position for the ABCDEA loop are:

$$\begin{aligned} f_1 &= r_B.\cos q + L_3.\cos\phi_1 + L_5.\cos\phi_2 + r_E.\cos(360° - q) - 60 \\ f_2 &= L_3.\sin\phi_1 + L_5.\sin\phi_2 \end{aligned} \tag{5.16}$$

The equations of the position for the ABCFEA loop are:

$$\begin{aligned} f_3 &= r_B.\cos q + L_3.\cos\phi_1 + L_6.\cos\phi_3 + L_7.\cos 240° - 60 \\ f_4 &= r_B.\sin q + L_3.\sin\phi_1 + L_6.\sin\phi_3 + L_7.\sin 240° \end{aligned} \tag{5.17}$$

$$\begin{aligned} \dot{f}_1 &= -r_B.\dot{q}.\sin q - L_3.\dot{\phi}_1.\sin\phi_1 - L_5.\dot{\phi}_2.\sin\phi_2 + r_E.\dot{q}.\sin(360° - q) \\ \dot{f}_2 &= L_3.\dot{\phi}_1.\cos\phi_1 + L_5.\dot{\phi}_2.\cos\phi_2 \\ \dot{f}_3 &= -r_B.\dot{q}.\sin q - L_3.\dot{\phi}_1.\sin\phi_1 - L_6.\dot{\phi}_3.\sin\phi_3 + \dot{L}_7.\cos 240° \\ \dot{f}_4 &= r_B.\dot{q}.\cos q + L_3.\dot{\phi}_1.\cos\phi_1 + L_6.\dot{\phi}_3.\cos\phi_3 + \dot{L}_7.\sin 240° \end{aligned} \tag{5.18}$$

$$\underbrace{\begin{bmatrix} L_3.\sin\phi_1 & -L_5.\sin\phi_2 & 0 & 0 \\ L_3.\cos\phi_1 & L_5.\cos\phi_2 & 0 & 0 \\ -L_3.\sin\phi_1 & 0 & -L_6.\sin\phi_3 & \cos 240° \\ -L_3.\cos\phi_1 & 0 & L_6.\cos\phi_3 & \sin 240° \end{bmatrix}}_{A} \cdot \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \\ \dot{\phi}_4 \end{bmatrix} = \begin{bmatrix} r_B.\sin q - r_E.\sin(360° - q) \\ 0 \\ r_B.\sin q \\ -r_B.\cos q \end{bmatrix} .\dot{q} \qquad (5.19)$$

$$\begin{aligned} &\ddot{f}_1 = -r_B.\dot{q}^2.\cos q - r_B.\ddot{q}.\sin q - L_3.\ddot{\phi}_1.\sin\phi_1 - L_3.\dot{\phi}_1^2.\cos\phi_1 - L_5.\ddot{\phi}_2.\sin\phi_2 - L_5.\dot{\phi}_2^2.\cos\phi_2 + \\ &+ r_E.\ddot{q}.\sin(360° - q) + r_E.\dot{q}^2.\cos(360° - q) \\ &\ddot{f}_2 = L_3.\ddot{\phi}_1.\cos\phi_1 - L_3.\dot{\phi}_1^2.\sin\phi_1 + L_5.\ddot{\phi}_2.\cos\phi_2 - L_5.\dot{\phi}_2^2.\sin\phi_2 \\ &\ddot{f}_3 = -r_B.\dot{q}^2.\cos q - r_B.\ddot{q}.\sin q - L_3.\ddot{\phi}_1.\sin\phi_1 - L_3.\dot{\phi}_1^2.\cos\phi_1 - L_6.\ddot{\phi}_3.\sin\phi_3 - L_6.\dot{\phi}_3^2.\cos\phi_3 + \ddot{L}_7.\cos 240° \\ &\ddot{f}_4 = -r_B.\dot{q}^2.\sin q + r_B.\ddot{q}.\cos q + L_3.\ddot{\phi}_1.\cos\phi_1 - L_3.\dot{\phi}_1^2.\sin\phi_1 - L_6.\dot{\phi}_3^2.\sin\phi_3 + L_6.\ddot{\phi}_3.\cos\phi_3 + \ddot{L}_7.\sin 240° \end{aligned} \qquad (5.20)$$

$$[A]\cdot \begin{bmatrix} \ddot{\phi}_1 \\ \ddot{\phi}_2 \\ \ddot{\phi}_3 \\ \ddot{\phi}_4 \end{bmatrix} = \begin{bmatrix} r_B.\dot{q}^2.\cos q + r_B.\ddot{q}.\sin q + L_3.\dot{\phi}_1^2.\cos\phi_1 + L_5.\dot{\phi}_2^2.\cos\phi_2 - r_E.\ddot{q}.\sin(360° - q) - r_E.\dot{q}^2.\cos(360° - q) \\ L_3.\dot{\phi}_1^2.\sin\phi_1 + L_5.\dot{\phi}_2^2.\sin\phi_2 \\ L_3.\dot{\phi}_1^2.\cos\phi_1 + L_6.\dot{\phi}_3^2.\cos\phi_3 \\ r_B.\dot{q}^2.\sin q - r_B.\ddot{q}.\cos q + L_3.\dot{\phi}_1^2.\sin\phi_1 + L_6.\dot{\phi}_3^2.\sin\phi_3 \end{bmatrix} \qquad (5.21)$$

The results from the kinematic analysis:

The specific solution was carried out on the basis of SolidWorks. In Fig. 5.16, there is the course of the rotational displacement of the 3, 5, 6 items (members) and moreover, the displacement of the F point in dependence on the rotational displacement of the crank, designated as 2, is shown in Fig. 5.17. In addition, the course of the angular speeds (velocities) of mentioned items (members) and the speeds (velocities) for the F point is shown in Figs. 5.18 and 5.19. The first four positions of the mechanism can be seen in Fig. 5.20.

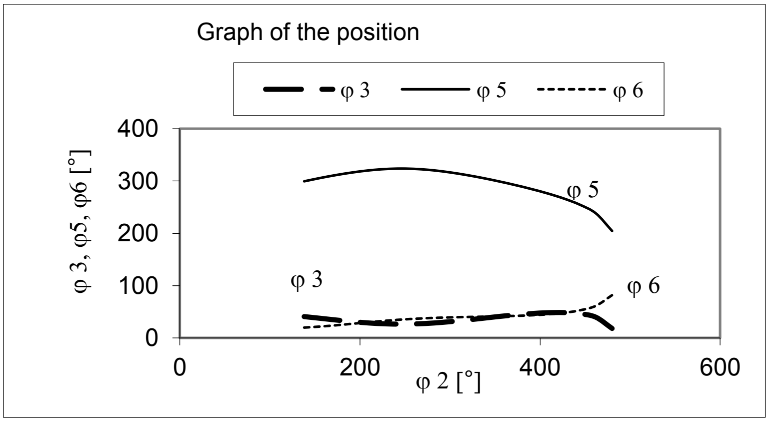


Fig. 5.16

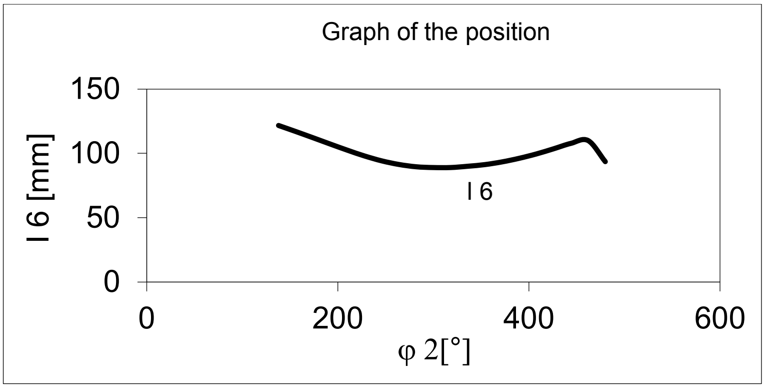


Fig. 5.17

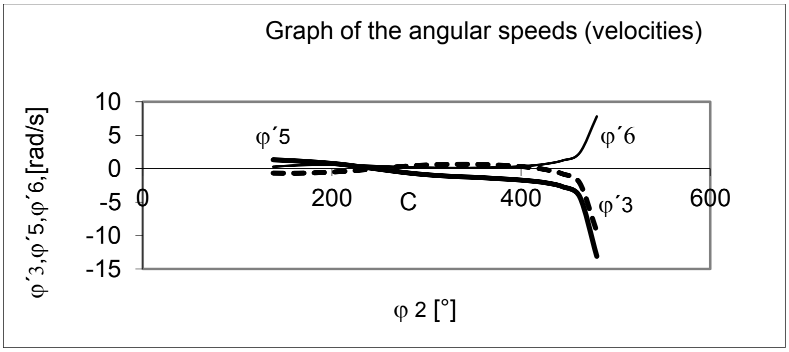


Fig. 5.18

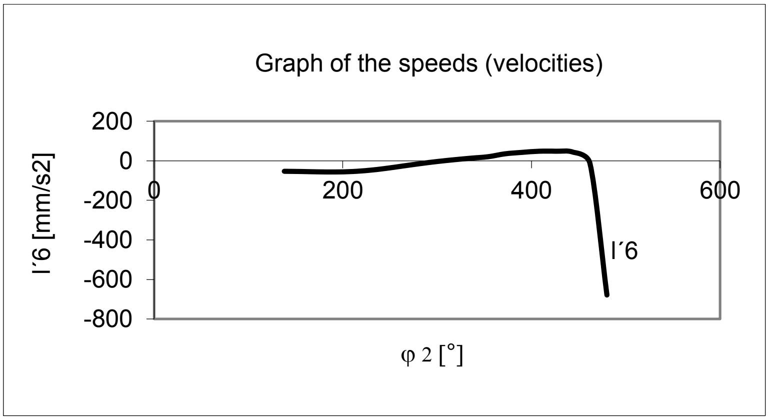
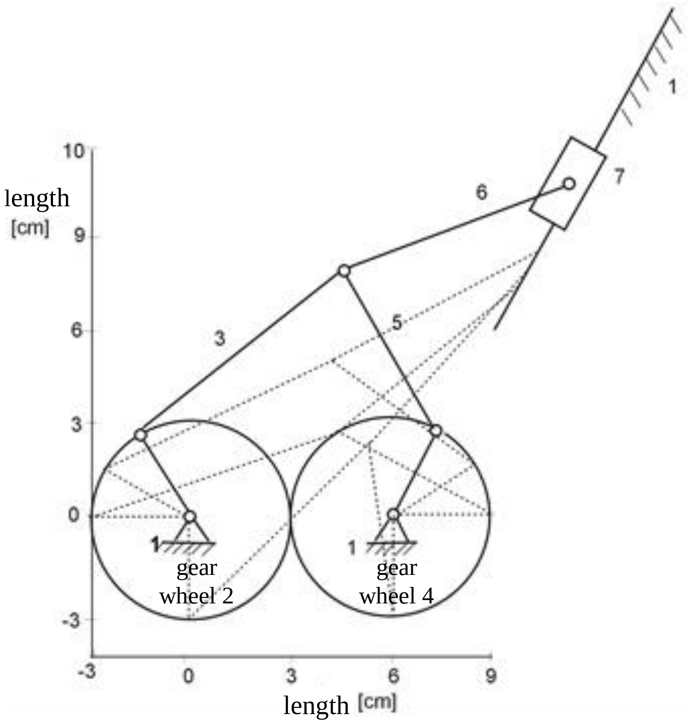


Fig. 5.19



Animation of the four positions of the mechanism

Fig. 5.20

5.3.1 Dynamic analysis of planar mechanism

The main objective of the dynamic analysis is connected with specification of the loading for the individual items (members) and determination of the courses relating to mutual reactions, referring to individual kinematic connections [4, 10–

12]. The analysis was based on utilisation of the nonlinear model. Relating to the analysis, the other important values were utilised:

- modulus of elasticity (Young's modulus): E = 210 GPa,
- Poisson's ratio: $\mu = 0.3$,
- density of material: ρ = 7850 kg m^{-3}.

Fig. 5.21 represents the course of the reaction in D point of the body, designated as 4 and Fig. 5.22 represents the course of the reaction in C point of the body, designated as 3.

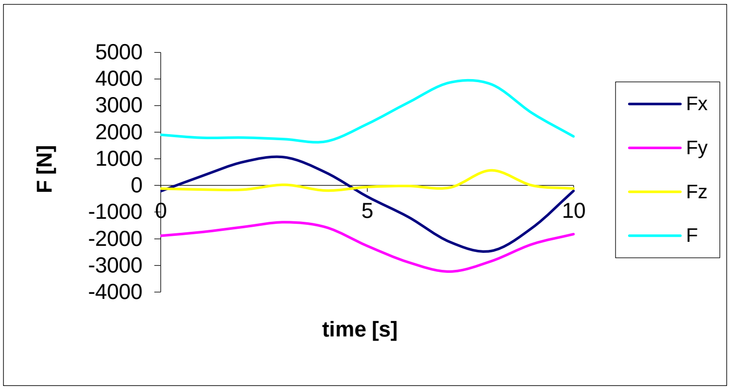


Fig. 5.21 Course of the reaction in D point of the body, designated as 4 – depending on time.

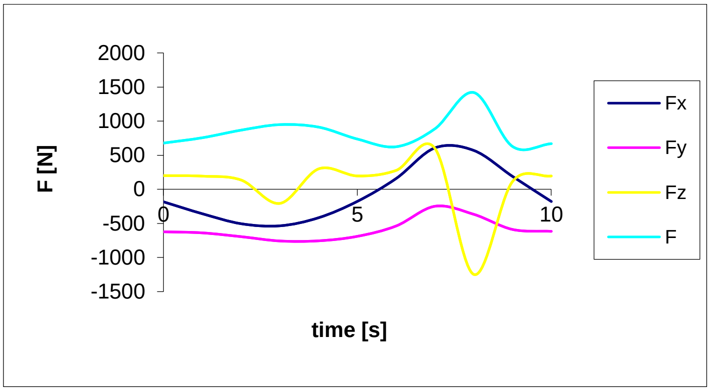


Fig. 5.22 Course of the reaction in C point of the body, designated as 3 – depending on time.

5.3.2 Distribution of the stress in items (members) of planar mechanism

The distribution of the stress for bodies [4,12-13], designated as 3, 6 can be seen in Fig. 5.23- Fig. 5.26.

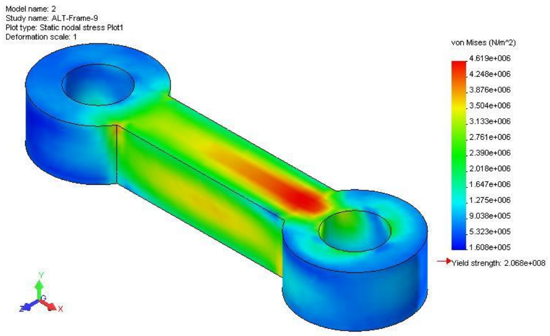


Fig. 5.23 Distribution of the stresses (Pa) for body, designated as 3.

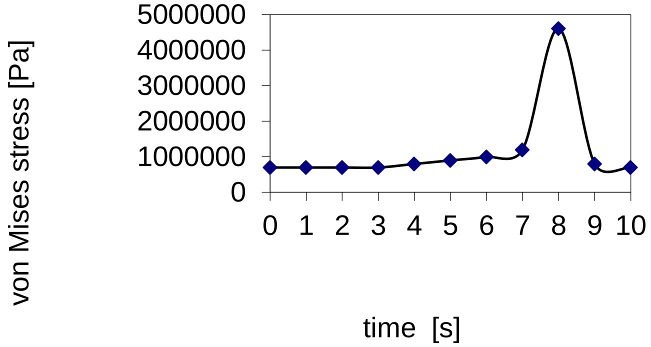


Fig. 5.24 Course of the stresses for body, designated as 3 – depending on time.

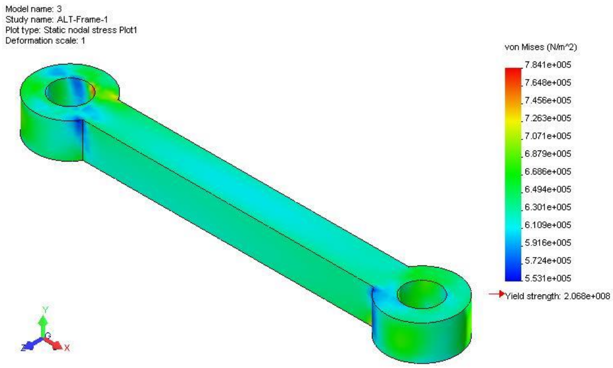


Fig. 5.25 Distribution of the stresses (Pa) for body, designated as 6.

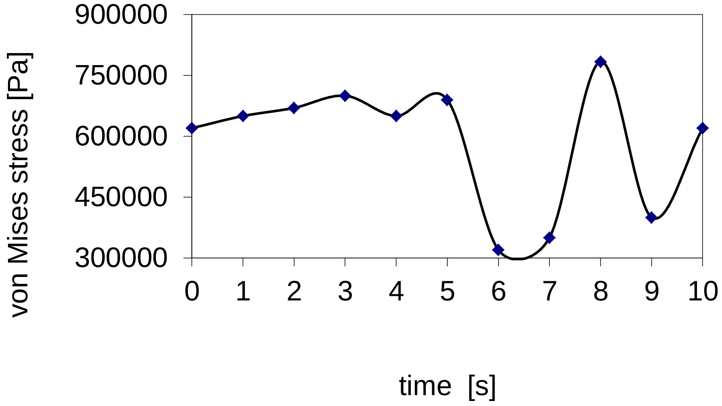


Fig. 5.26 Course of the stresses for body, designated as 6 – depending on time.

5.4 Kinematic Analysis for Ten-Item Mechanism

Considering the ten-item planar mechanism, the vector method [28],[40] is used for solution relating to field of positions, velocities and accelerations. The numerical solution procedures are carried out for predefined input parameters or values (5.22) relating to the Stirling engine, which can be seen in Fig. 5.27. The kinematic scheme can be seen in the Fig. 5.28. The equations of position (5.24) are differentiated on the basis of $\phi_i(i=1,8)$ variables and it leads to the obtaining of the equations of velocities (5.25). Subsequently, the given equations of velocities are transformed to matrix form (5.36). If the equations (5.25) are differentiated once more, the equations of accelerations (5.26) are obtained and after that, they are transformed to the matrix form (5.37). The equations from (5.27) up to (5.35) are valid and used for calculation of kinematic values of distance, speed (velocity) and acceleration for D, K, J points. The given solution procedures are carried out by means of the Nastran software and results of kinematic analysis are shown in Figs. from 5.29 to 5.35.

Input parameters:

$$
\begin{aligned}
&q = \Theta_1 = 2{,}0864\ rad && a_{51} = c_{14} = 6{,}0\ cm \\
&\Theta_2 = \phi_1 = 1{,}2745\ rad && a_{52} = c_{15} = 5{,}0\ cm \\
&r_3 = \phi_2 = 7{,}0\ cm && a_6 = c_{16} = 2{,}4\ cm \\
&\Theta_4 = \phi_3 = 0{,}3818\ rad && a_8 = c_{17} = 4{,}8\ cm \\
&\Theta_5 = \phi_4 = 0{,}2101\ rad && L_1 = c_{18} = 9{,}4\ cm \\
&\Theta_6 = \phi_5 = 0{,}2003\ rad && L_2 = c_{19} = 6{,}8\ cm \\
&r_7 = \phi_6 = 2{,}4\ cm && L_3 = c_{20} = 1{,}9\ cm \\
&\Theta_8 = \phi_7 = 1{,}0973\ rad && L_4 = c_{21} = 7{,}0\ cm \\
&r_9 = \phi_8 = 1{,}7\ cm && L_5 = c_{22} = 7{,}0\ cm \\
&a_1 = c_9 = 2{,}4\ cm && L_6 = c_{23} = 6{,}0\ cm \\
&a_{21} = c_{10} = 5{,}0\ cm && \alpha = c_{24} = 0{,}4907\ rad \\
&a_{22} = c_{11} = 1{,}0\ cm && \beta = c_{25} = 0{,}6338\ rad \\
&b = c_{12} = 0{,}4\ cm && \gamma = c_{26} = 1{,}6258\ rad \\
&a_4 = c_{13} = 5{,}1\ cm && \delta = c_{27} = 0{,}1411\ rad \\
&\omega_{21} = 1\ rad.s^{-1} && \alpha_{21} = 0\ rad.s^{-2}
\end{aligned}
\tag{5.22}
$$

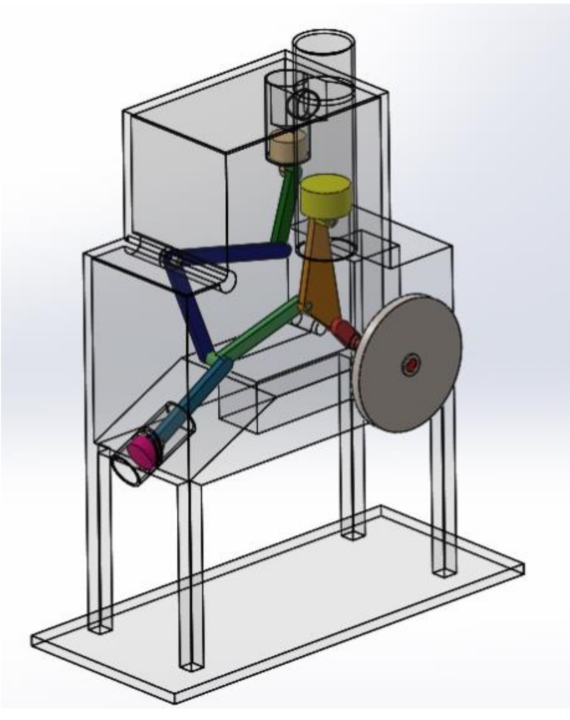


Fig. 5.27 The computational model of the Stirling engine

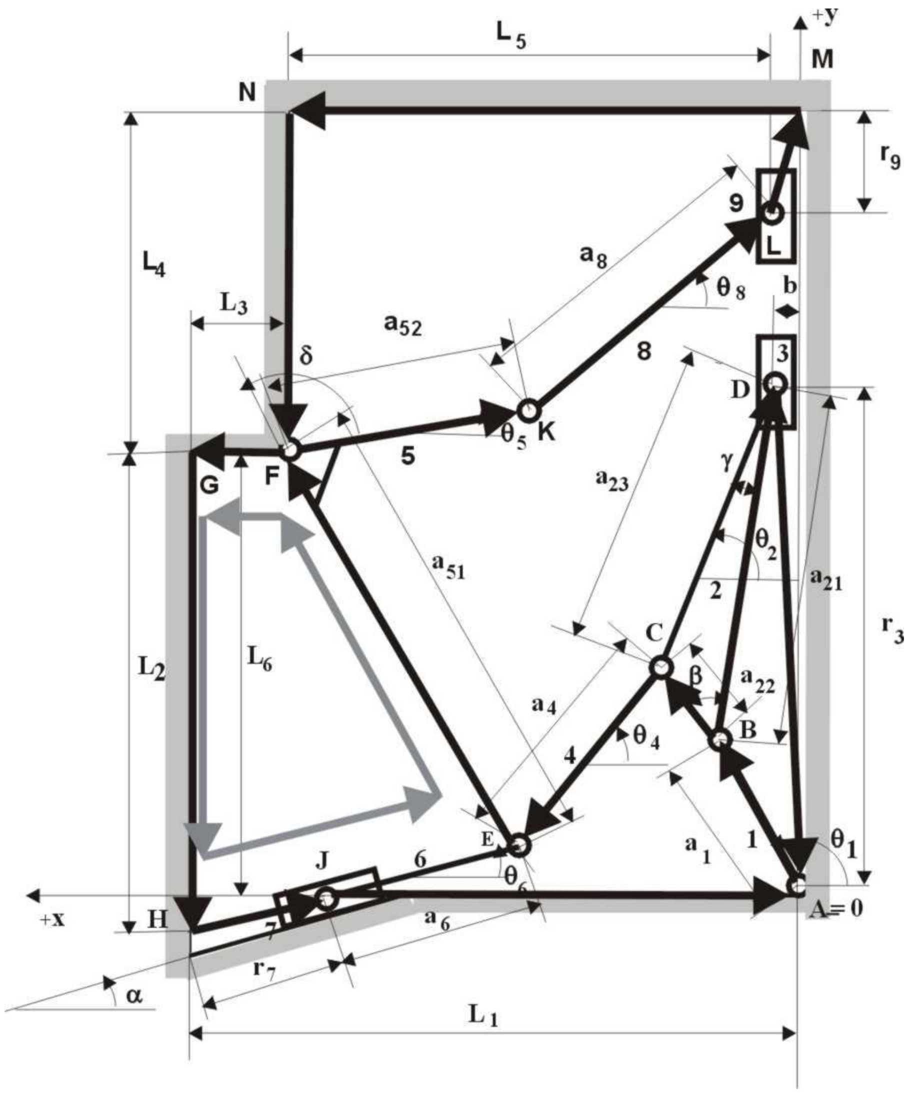


Fig. 5.28 Kinematic scheme of mechanism

The variables are:

$$[\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6, \phi_7, \phi_8] = [\Theta_2, r_3, \Theta_4, \Theta_5, \Theta_6, r_7, \Theta_8, r_9] \tag{5.23}$$

Considering ABDA, ABCEFGHJA, EFGHJE and FKLMNF, the equations of the position for the loop are:

$$
\begin{aligned}
&f_1 = -b - a_{21}\cos(\phi_1 - \gamma) - a_1 \cos q = 0\\
&f_2 = -\phi_2 + a_{21}\sin(\phi_1 - \gamma) + a_1 \sin q = 0\\
&f_3 = -a_1 \cos q - a_{22}\cos(\phi_1 - \gamma + \beta) + a_4 \cos\phi_3 - a_{51}\cos(\phi_4 + \delta) + L_3 - L_1 = 0\\
&f_4 = a_1 \sin q + a_{22}\sin(\phi_1 - \gamma + \beta) - a_4 \sin\phi_3 + a_{51}\sin(\phi_4 + \delta) - L_6 = 0\\
&f_5 = -\phi_6 \cos\alpha - a_6 \cos\phi_5 - a_{51}\cos(\phi_4 + \delta) + L_3 = 0\\
&f_6 = \phi_6 \sin\alpha + a_6 \sin\phi_5 + a_{51}\sin(\phi_4 + \delta) - L_2 = 0\\
&f_7 = -a_{52}\cos\phi_4 - a_8 \cos\phi_7 + L_5 = 0\\
&f_8 = a_{52}\sin\phi_4 + a_8 \sin\phi_7 + \phi_8 - L_4 = 0
\end{aligned} \tag{5.24}
$$

If the equations of the positions (5.24) are differentiated on the basis of $\phi_i(i = 1, 8)$ variables, the system of velocity equations (5.25) is obtained and after further differentiation, the system of acceleration equations (5.26) can be obtained.

$$
\begin{aligned}
&\dot{f}_1 = a_{21}\dot{\phi}_1 \sin(\phi_1 - \gamma) = -a_1\dot{q}\sin q\\
&\dot{f}_2 = -\dot{\phi}_2 + a_{21}\dot{\phi}_1 \cos(\phi_1 - \gamma) = -a_1\dot{q}\cos q\\
&\dot{f}_3 = a_{22}\dot{\phi}_1 \sin(\phi_1 - \gamma + \beta) - a_4\dot{\phi}_3 \sin\phi_3 + a_{51}\dot{\phi}_4 \sin(\phi_4 + \delta) = -a_1\dot{q}\sin q\\
&\dot{f}_4 = a_{22}\dot{\phi}_1 \cos(\phi_1 - \gamma + \beta) - a_4\dot{\phi}_3 \cos\phi_3 + a_{51}\dot{\phi}_4 \cos(\phi_4 + \delta) = -a_1\dot{q}\cos q\\
&\dot{f}_5 = -\dot{\phi}_6 \cos\alpha + a_6\dot{\phi}_5 \sin\phi_5 + a_{51}\dot{\phi}_4 \sin(\phi_4 + \delta) = 0\\
&\dot{f}_6 = \dot{\phi}_6 \sin\alpha + a_6\dot{\phi}_5 \cos\phi_5 + a_{51}\dot{\phi}_4 \cos(\phi_4 + \delta) = 0\\
&\dot{f}_7 = a_{52}\dot{\phi}_4 \sin\phi_4 + a_8\dot{\phi}_7 \sin\phi_7 = 0\\
&\dot{f}_8 = a_{52}\dot{\phi}_4 \cos\phi_4 + a_8\dot{\phi}_7 \cos\phi_7 + \dot{\phi}_8 = 0
\end{aligned} \tag{5.25}
$$

$$
\begin{aligned}
&\ddot{f}_1 = a_{21}\ddot{\phi}_1 \sin(\phi_1 - \gamma) + a_{21}\dot{\phi}_1^2 \cos(\phi_1 - \gamma) = -a_1\ddot{q}\sin q - a_1\dot{q}^2 \cos q\\
&\ddot{f}_2 = -\ddot{\phi}_2 + a_{21}\ddot{\phi}_1 \cos(\phi_1 - \gamma) - a_{21}\dot{\phi}_1^2 \sin(\phi_1 - \gamma) = -a_1\ddot{q}\cos q + a_1\dot{q}^2 \sin q\\
&\ddot{f}_3 = a_{22}\ddot{\phi}_1 \sin(\phi_1 - \gamma + \beta) + a_{22}\dot{\phi}_1^2 \cos(\phi_1 - \gamma + \beta) - a_4\ddot{\phi}_3 \sin\phi_3 - a_4\dot{\phi}_3^2 \cos\phi_3 +\\
&+ a_{51}\ddot{\phi}_4 \sin(\phi_4 + \delta) + a_{51}\dot{\phi}_4^2 \cos(\phi_4 + \delta) = -a_1\ddot{q}\sin q - a_1\dot{q}^2 \cos q\\
&\ddot{f}_4 = a_{22}\ddot{\phi}_1 \cos(\phi_1 - \gamma + \beta) - a_{22}\dot{\phi}_1^2 \sin(\phi_1 - \gamma + \beta) - a_4\ddot{\phi}_3 \cos\phi_3 +\\
&+ a_4\dot{\phi}_3^2 \sin\phi_3 + a_{51}\ddot{\phi}_4 \cos(\phi_4 + \delta) - a_{51}\dot{\phi}_4^2 \sin(\phi_4 + \delta) = -a_1\ddot{q}\cos q + a_1\dot{q}^2 \sin q\\
&\ddot{f}_5 = -\ddot{\phi}_6 \cos\alpha + a_6\ddot{\phi}_5 \sin\phi_5 + a_6\dot{\phi}_5^2 \cos\phi_5 + a_{51}\ddot{\phi}_4 \sin(\phi_4 + \delta) + a_{51}\dot{\phi}_4^2 \cos(\phi_4 + \delta) = 0\\
&\ddot{f}_6 = \ddot{\phi}_6 \sin\alpha + a_6\ddot{\phi}_5 \cos\phi_5 - a_6\dot{\phi}_5^2 \sin\phi_5 + a_{51}\ddot{\phi}_4 \cos(\phi_4 + \delta) - a_{51}\dot{\phi}_4^2 \sin(\phi_4 + \delta) = 0\\
&\ddot{f}_7 = -a_{52}\ddot{\phi}_4 \sin\phi_4 - a_{52}\dot{\phi}_4^2 \cos\phi_4 - a_8\ddot{\phi}_7 \sin\phi_7 - a_8\dot{\phi}_7^2 \cos\phi_7 = 0\\
&\ddot{f}_8 = a_{52}\ddot{\phi}_4 \cos\phi_4 - a_{52}\dot{\phi}_4^2 \sin\phi_4 + a_8\ddot{\phi}_7 \cos\phi_7 - a_8\dot{\phi}_7^2 \sin\phi_7 + \ddot{\phi}_8 = 0
\end{aligned} \tag{5.26}
$$

The equations from (5.27) up to (5.35) are valid and used for calculation of kinematic values of distance, velocity and acceleration for D, K, J points.

$$x_D = b$$
$$y_D = a_1 \sin q + a_{21} \sin(\phi_1 - \gamma) = \phi_2 \tag{5.27}$$

$$\dot{x}_D = 0$$
$$\dot{y}_D = a_1.\dot{q} \cos q + a_{21}.\dot{\phi}_1 \cos(\phi_1 - \gamma) = \dot{\phi}_2 \tag{5.28}$$

$$\ddot{x}_D = 0$$
$$\ddot{y}_D = a_1.\ddot{q} \cos q - a_1.\dot{q}^2 \sin q + a_{21}.\ddot{\phi}_1 \cos(\phi_1 - \gamma) - a_{21}.\dot{\phi}^2{}_1 \sin(\phi_1 - \gamma) = \ddot{\phi}_2 \tag{5.29}$$

$$\begin{aligned} x_L &= b \\ y_L &= a_1 \sin q + a_{22} \sin(\phi_1 - \gamma + \beta) - a_4 \sin\phi_3 + a_{51} \sin(\phi_4 + \delta) + a_{52} \sin\phi_4 + \\ &+ a_8 \sin\phi_7 = L_6 + L_4 - \phi_8 \end{aligned} \tag{5.30}$$

$$\begin{aligned} \dot{x}_L &= 0 \\ \dot{y}_L &= a_1.\dot{q} \cos q + a_{22}.\dot{\phi}_1 \cos(\phi_1 - \gamma + \beta) - a_4.\dot{\phi}_3 \cos\phi_3 + a_{51}.\dot{\phi}_4 \cos(\phi_4 + \delta) + \\ &+ a_{52}.\dot{\phi}_4 \cos\phi_4 + a_8.\dot{\phi}_7 \cos\phi_7 = -\dot{\phi}_8 \end{aligned} \tag{5.31}$$

$$\begin{aligned} \ddot{x}_L &= 0 \\ \ddot{y}_L &= a_1.\ddot{q} \cos q - a_1.\dot{q}^2 \sin q + a_{22}.\ddot{\phi}_1 \cos(\phi_1 - \gamma + \beta) - a_{22}.\ddot{\phi}_1^2 \sin(\phi_1 - \gamma + \beta) - \\ &- a_4.\ddot{\phi}_3 \cos\phi_3 + a_4.\dot{\phi}_3^2 \sin\phi_3 + a_{51}.\ddot{\phi}_4 \cos(\phi_4 + \delta) - a_{51}.\dot{\phi}_4^2 \sin(\phi_4 + \delta) + \\ &+ a_{52}.\ddot{\phi}_4 \cos\phi_4 - a_{52}.\dot{\phi}_4^2 \sin\phi_4 + a_8.\ddot{\phi}_7 \cos\phi_7 - a_8.\dot{\phi}_7^2 \sin\phi_7 = -\ddot{\phi}_8 \end{aligned} \tag{5.32}$$

$$\begin{aligned} x_J &= -a_1 \cos q - a_{22} \cos(\phi_1 - \gamma + \beta) + a_4 \cos\phi_3 + a_6 \cos\phi_5 = L_1 - \phi_6 \cos\alpha \\ y_J &= a_1 \sin q + a_{22} \sin(\phi_1 - \gamma + \beta) - a_4 \sin\phi_3 - a_6 \sin\phi_5 = L_6 - L_2 + \phi_6 \cos\alpha \end{aligned} \tag{5.33}$$

$$\begin{aligned} \dot{x}_J &= a_1.\dot{q} \sin q + a_{22}.\dot{\phi}_1 \sin(\phi_1 - \gamma + \beta) - a_4.\dot{\phi}_3 \sin\phi_3 - a_6.\dot{\phi}_5 \sin\phi_5 = -\dot{\phi}_6 \cos\alpha \\ \dot{y}_J &= a_1.\dot{q} \cos q + a_{22}.\dot{\phi}_1 \cos(\phi_1 - \gamma + \beta) - a_4.\dot{\phi}_3 \cos\phi_3 - a_6.\dot{\phi}_5 \cos\phi_5 = \dot{\phi}_6 \sin\alpha \end{aligned} \tag{5.34}$$

$$\begin{aligned} \ddot{x}_J &= a_1.\ddot{q} \sin q + a_1.\dot{q}^2 \cos q + a_{22}.\ddot{\phi}_1 \sin(\phi_1 - \gamma + \beta) + a_{22}.\dot{\phi}_1^2 \cos(\phi_1 - \gamma + \beta) - \\ &- a_4.\ddot{\phi}_3 \sin\phi_3 - a_4.\dot{\phi}_3^2 \cos\phi_3 - a_6.\ddot{\phi}_5 \sin\phi_5 - a_6.\dot{\phi}_5^2 \cos\phi_5 = -\ddot{\phi}_6 \cos\alpha \\ \ddot{y}_J &= a_1.\ddot{q} \cos q - a_1.\dot{q}^2 \sin q + a_{22}.\ddot{\phi}_1 \cos(\phi_1 - \gamma + \beta) - a_{22}.\dot{\phi}_1^2 \sin(\phi_1 - \gamma + \beta) - \\ &- a_4.\ddot{\phi}_3 \cos\phi_3 + a_4.\dot{\phi}_3^2 \sin\phi_3 - a_6.\ddot{\phi}_5 \cos\phi_5 + a_6.\dot{\phi}_5^2 \sin\phi_5 = \ddot{\phi}_6 \sin\alpha \end{aligned} \tag{5.35}$$

The system of equations (5.25) is transformed to matrix form (5.36) as well as the system of equations (5.26) is transformed to the matrix form (5.37).

$$\begin{bmatrix} a_{21}\sin(\phi_1-\gamma) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21}\cos(\phi_1-\gamma) & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{22}\sin(\phi_1-\gamma+\beta) & 0 & -a_4\sin\phi_3 & +a_{51}\sin(\phi_4+\delta) & 0 & 0 & 0 & 0 \\ a_{22}\cos(\phi_1-\gamma+\beta) & 0 & -a_4\cos\phi_3 & a_{51}\cos(\phi_4+\delta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +a_{51}\sin(\phi_4+\delta) & a_6\sin\phi_5 & -\cos\alpha & 0 & 0 \\ 0 & 0 & 0 & a_{51}\cos(\phi_4+\delta) & a_6\cos\phi_5 & \sin\alpha & 0 & 0 \\ 0 & 0 & 0 & a_{52}\sin\phi_4 & 0 & 0 & a_8\sin\phi_7 & 0 \\ 0 & 0 & 0 & a_{52}\cos\phi_4 & 0 & 0 & a_8\cos\phi_7 & 1 \end{bmatrix}\cdot \tag{5.36}$$

$$\cdot\begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \\ \dot{\phi}_4 \\ \dot{\phi}_5 \\ \dot{\phi}_6 \\ \dot{\phi}_7 \\ \dot{\phi}_8 \end{bmatrix} = \begin{bmatrix} -a_1\dot{q}\sin q \\ -a_1\dot{q}\cos q \\ -a_1\dot{q}\sin q \\ -a_1\dot{q}\cos q \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a_{21}\sin(\phi_1-\gamma) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21}\cos(\phi_1-\gamma) & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{22}\sin(\phi_1-\gamma+\beta) & 0 & -a_4\sin\phi_3 & a_{51}\sin(\phi_4+\delta) & 0 & 0 & 0 & 0 \\ a_{22}\cos(\phi_1-\gamma+\beta) & 0 & -a_4\cos\phi_3 & a_{51}\cos(\phi_4+\delta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{51}\sin(\phi_4+\delta) & a_6\sin\phi_5 & -\cos\alpha & 0 & 0 \\ 0 & 0 & 0 & a_{51}\cos(\phi_4+\delta) & a_6\cos\phi_5 & \sin\alpha & 0 & 0 \\ 0 & 0 & 0 & a_{52}\sin\phi_4 & 0 & 0 & a_8\sin\phi_7 & 0 \\ 0 & 0 & 0 & a_{52}\cos\phi_4 & 0 & 0 & a_8\cos\phi_7 & 1 \end{bmatrix}\cdot\begin{bmatrix} \ddot{\phi}_1 \\ \ddot{\phi}_2 \\ \ddot{\phi}_3 \\ \ddot{\phi}_4 \\ \ddot{\phi}_5 \\ \ddot{\phi}_6 \\ \ddot{\phi}_7 \\ \ddot{\phi}_8 \end{bmatrix} =$$

$$= \begin{bmatrix} -a_{21}\dot{\phi}_1^2\cos(\phi_1-\gamma) - a_1\ddot{q}\sin q - a_1\dot{q}^2\cos q \\ a_{21}\dot{\phi}_1^2\sin(\phi_1-\gamma) - a_1\ddot{q}\cos q + a_1\dot{q}^2\sin q \\ a_{22}\dot{\phi}_1^2\cos(\phi_1-\gamma+\beta) + a_4\dot{\phi}_3^2\cos\phi_3 - a_{51}\dot{\phi}_4^2\cos(\phi_4+\delta) - a_1\ddot{q}\sin q - a_1\dot{q}^2\cos q \\ a_{22}\dot{\phi}_1^2\sin(\phi_1-\gamma+\beta) - a_4\dot{\phi}_3^2\sin\phi_3 + a_{51}\dot{\phi}_4^2\sin(\phi_4+\delta) - a_1\ddot{q}\cos q + a_1\dot{q}^2\sin q \\ -a_6\dot{\phi}_5^2\cos\phi_5 - a_{51}\dot{\phi}_4^2\cos(\phi_4+\delta) \\ a_6\dot{\phi}_5^2\sin\phi_5 + a_{51}\dot{\phi}_4^2\sin(\phi_4+\delta) \\ -a_{52}\dot{\phi}_4^2\cos\phi_4 - a_8\dot{\phi}_7^2\cos\phi_7 \\ a_{52}\dot{\phi}_4^2\sin\phi_4 + a_8\dot{\phi}_7^2\sin\phi_7 \end{bmatrix} \tag{5.37}$$

The results from the kinematic analysis

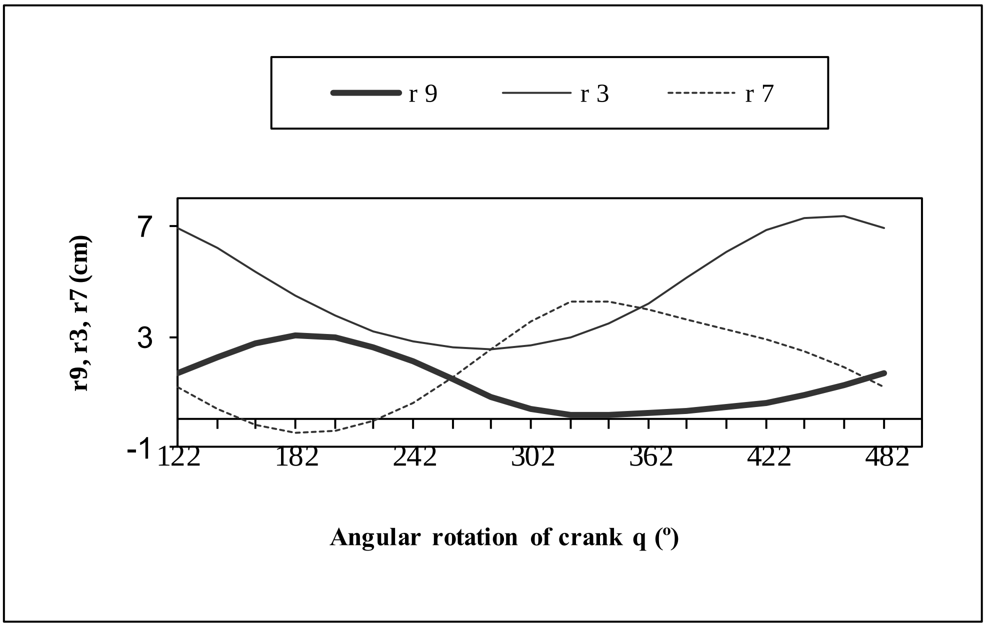


Fig. 5.29 The course of displacements for D, L, J points in dependence on angular rotation of crank

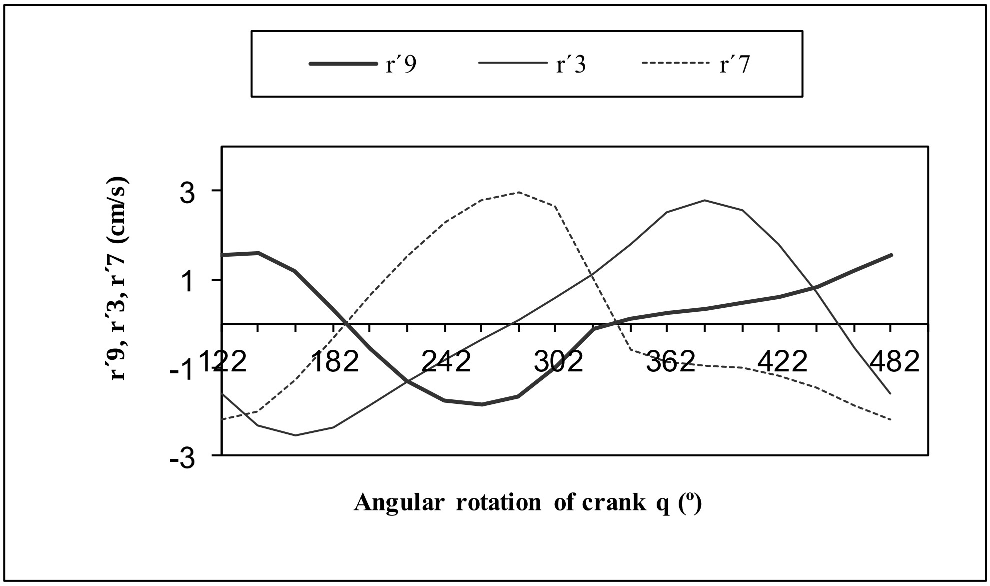


Fig. 5.30 The course of velocity for D, L, J points in dependence on angular rotation of crank

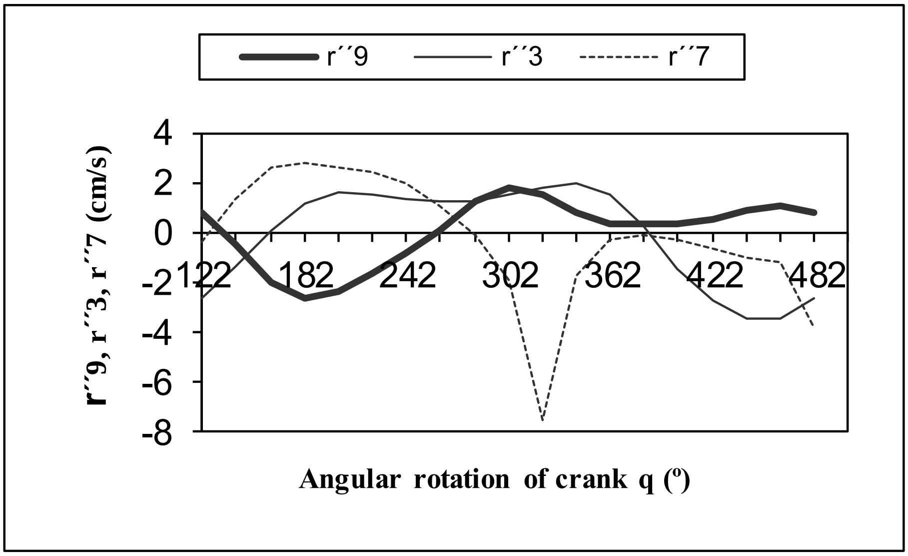


Fig. 5.31 The course of acceleration for D, L, J points in dependence on angular rotation of crank

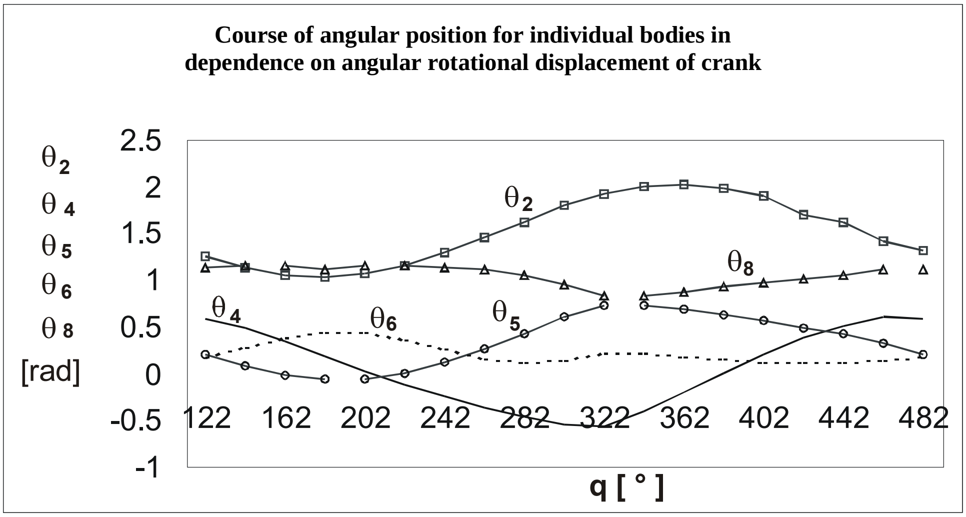


Fig. 5.32 The course of angular position for individual bodies in dependence on angular rotation of crank

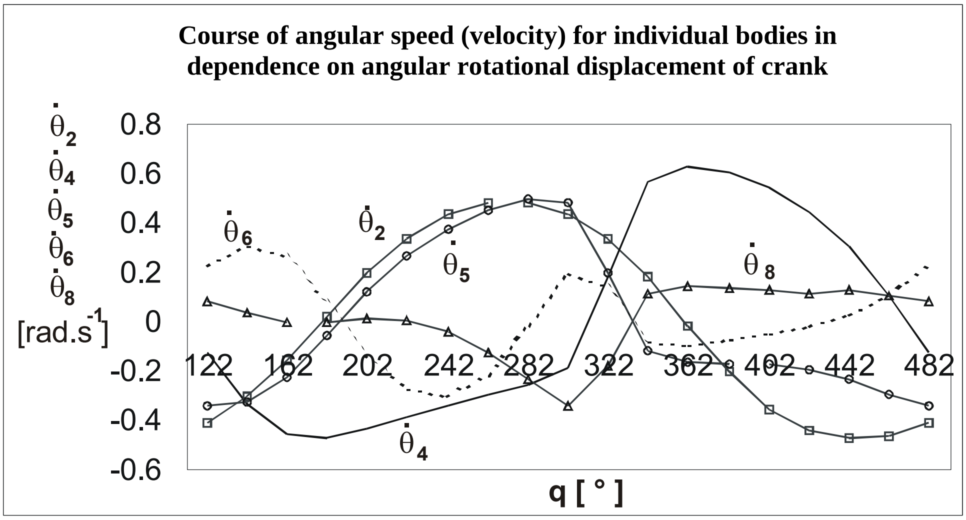


Fig. 5.33 The course of angular velocity for individual bodies in dependence on angular rotation of crank

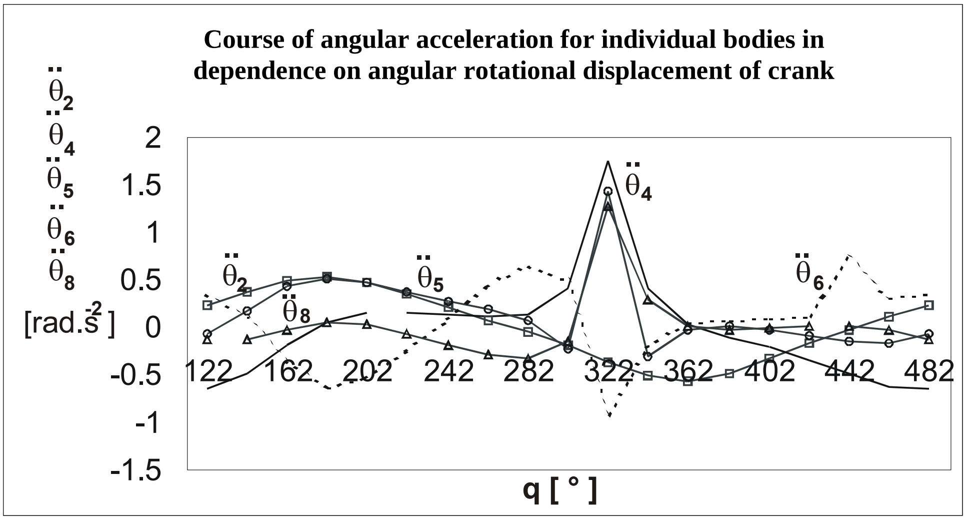
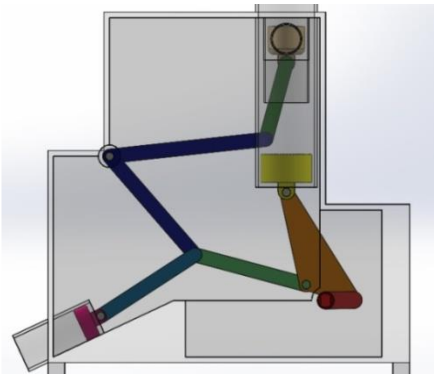
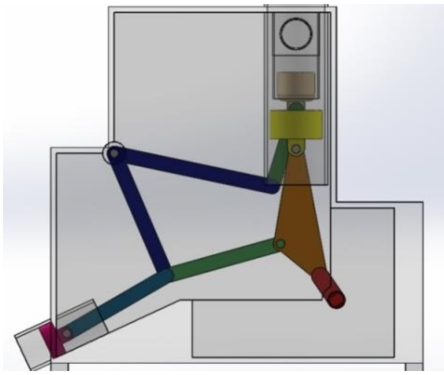


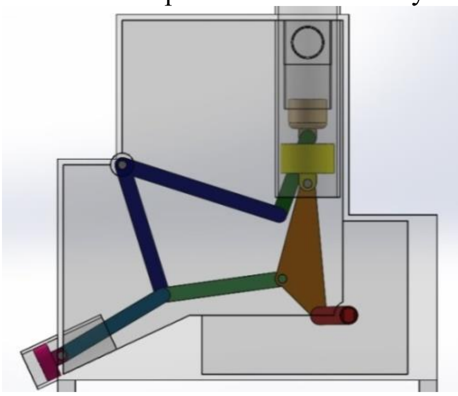
Fig. 5.34 The course of angular acceleration for individual bodies in dependence on angular rotation of crank



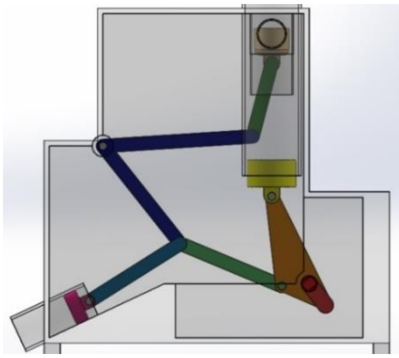
Rotational displacement of crank by 0°



Rotational displacement of crank by 120°



Rotational displacement of crank by 180°



Rotational displacement of crank by 300°

Fig. 5.35 Animation of the motion (movement) of the mechanism

5.5 Kinematic Analysis of the Pressing Machine

In relation to the six-item mechanism [39] (mechanism with the six members), the field of positions, speeds (velocities) and accelerations is going to be solved by the vector method. The numerical solution is carried out for the specified input values (5.38) if the specified angle of rotational displacement for the driving item (member), designated as **2,** is: $q=\varphi_2=315^\circ$ and the revolutions per minutes are: *n* = 3.5 rpm. The mechanism represents a pressing machine (Fig. 5.36), the kinematic scheme of which can be seen in Fig. 5.37. Formation of the equations of the position (5.40), (5.41) the differentiation of which, for $\phi_i(i=1,2,3,4)$ variables, gives the specific equations for the speeds (velocities) (5.42) and then the given equations are rewritten into a matrix form (5.44). If the equations (5.42) are differentiated once more, we can get equations (5.43) for the accelerations and the given equation can be rewritten into a matrix form (5.45).

To simplify the numerical solution, the solution can be carried out by help of the submatrix equations (5.46) and (5.47). The given solution was carried out by using the SolidWorks and the results of the kinematic analysis are shown in Figs. 5.38, 5.39, 5.40 and the animation for the first four positions is shown in Fig. 5.41.

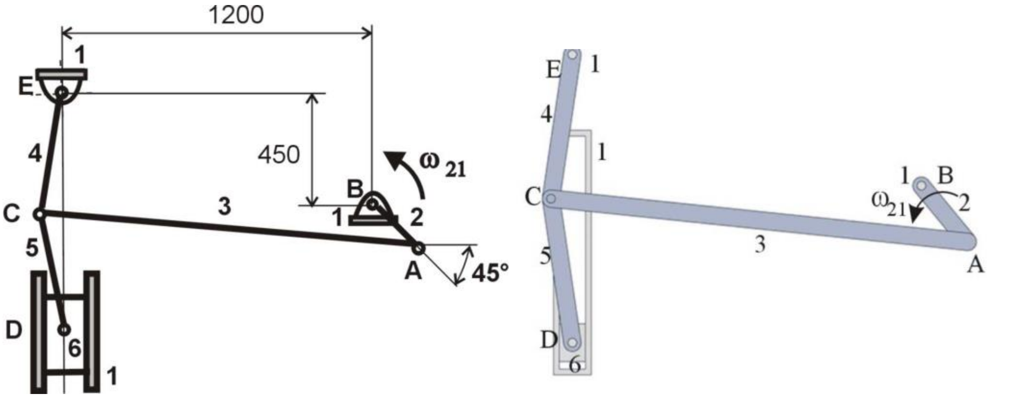


Fig. 5.36 Planar mechanism – computational model.

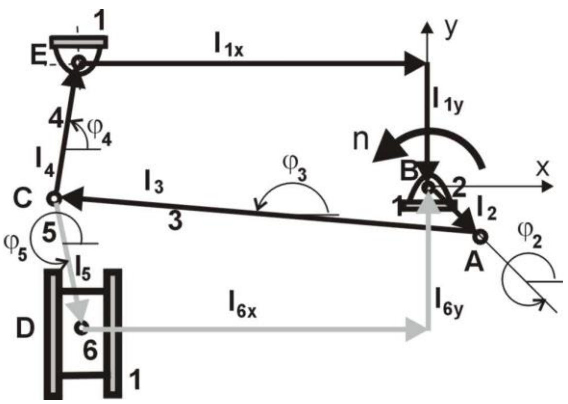


Fig. 5.37 Kinematic scheme of the given mechanism

Input parameters:

$$BA = l_2 = 0.25\ m,\ AC = l_3 = 1.44\ m,\ CE = l_4 = 0.5\ m,\ CD = l_5 = 0.5\ m,$$
$$q = \varphi_2 = 315°, \varphi_3 = 10^\circ, \varphi_4 = 85^\circ, \varphi_5 = 275^\circ, n = 3.5\ ot/\min. \quad (5.38)$$

The variables are:

$$[\phi_1, \phi_2, \phi_3, \phi_4] = [\varphi_3, \varphi_4, \varphi_5, l_{6y}] \tag{5.39}$$

The equations of the position for the BACEB loop are:

$$\begin{aligned} f_1 &= l_2 \cos\varphi_2 + l_3 \cos\varphi_3 + l_4 \cos\varphi_4 + l_{1x} = 0, \\ f_2 &= l_2 \sin\varphi_2 + l_3 \sin\varphi_3 + l_4 \sin\varphi_4 - l_{1y} = 0. \end{aligned} \tag{5.40}$$

The BE vector can be rewritten into two vectors in the direction of the coordinate axes. The position equations for the BACDB loop are:

$$\begin{aligned} f_3 &= l_2 \cos\varphi_2 + l_3 \cos\varphi_3 + l_5 \cos\varphi_5 + l_{6x} = 0, \\ f_4 &= l_2 \sin\varphi_2 + l_3 \sin\varphi_3 + l_5 \sin\varphi_5 + l_{6y} = 0. \end{aligned} \tag{5.41}$$

The BD vector is rewritten into two vectors in the direction of the l_{6y}, l_{6x} coordinate axes:

$$\begin{aligned} \dot f_1 &= -l_2.\dot\varphi_2.\sin\varphi_2 - l_3.\dot\varphi_3 \sin\varphi_3 - l_4\dot\varphi_4 \sin\varphi_4 = 0, \\ \dot f_2 &= l_2.\dot\varphi_2.\cos\varphi_2 + l_3.\dot\varphi_3.\cos\varphi_3 + l_4.\dot\varphi_4.\cos\varphi_4 = 0, \\ \dot f_3 &= -l_2.\dot\varphi_2.\sin\varphi_2 - l_3.\dot\varphi_3 \sin\varphi_3 - l_5\dot\varphi_5 \sin\varphi_5 = 0, \\ \dot f_4 &= l_2.\dot\varphi_2.\cos\varphi_2 + l_3.\dot\varphi_3.\cos\varphi_3 + l_5.\dot\varphi_5.\cos\varphi_5 + \dot l_{6y} = 0 \end{aligned} \tag{5.42}$$

$$\begin{aligned} \ddot f_1 &= -l_2.\dot\varphi_2^2.\cos\varphi_2 - l_2.\ddot\varphi_2.\sin\varphi_2 - l_3.\dot\varphi_3^2 \cos\varphi_3 - l_3.\ddot\varphi_3 \sin\varphi_3 - l_4\dot\varphi_4^2 \cos\varphi_4 - l_4\ddot\varphi_4 \sin\varphi_4 = 0, \\ \ddot f_2 &= -l_2.\dot\varphi_2^2.\sin\varphi_2 + l_2.\ddot\varphi_2.\cos\varphi_2 - l_3.\dot\varphi_3^2 \sin\varphi_3 + l_3.\ddot\varphi_3 \cos\varphi_3 - l_4\dot\varphi_4^2 \sin\varphi_4 + l_4\ddot\varphi_4 \cos\varphi_4 = 0, \\ \ddot f_3 &= -l_2.\dot\varphi_2^2.\cos\varphi_2 - l_2.\ddot\varphi_2.\sin\varphi_2 - l_3.\dot\varphi_3^2 \cos\varphi_3 - l_3.\ddot\varphi_3 \sin\varphi_3 - l_5\dot\varphi_5^2 \cos\varphi_5 - l_5\ddot\varphi_5 \sin\varphi_5 = 0, \\ \ddot f_4 &= -l_2.\dot\varphi_2^2.\sin\varphi_2 + l_2.\ddot\varphi_2.\cos\varphi_2 - l_3.\dot\varphi_3^2 \sin\varphi_3 + l_3.\ddot\varphi_3 \cos\varphi_3 - l_5\dot\varphi_5^2 \sin\varphi_5 + l_5\ddot\varphi_5 \cos\varphi_5 + \ddot l_{6y} = 0 \end{aligned} \tag{5.43}$$

$$\begin{bmatrix} -l_3\sin\varphi_3 & -l_4\sin\varphi_4 & 0 & 0 \\ l_3\cos\varphi_3 & l_4\cos\varphi_4 & 0 & 0 \\ -l_3\sin\varphi_3 & 0 & -l_5\sin\varphi_5 & 0 \\ l_3\cos\varphi_3 & 0 & l_5\cos\varphi_5 & 1 \end{bmatrix} . \begin{bmatrix} \dot\varphi_3 \\ \dot\varphi_4 \\ \dot\varphi_5 \\ \dot l_{6y} \end{bmatrix} = \begin{bmatrix} l_2.\sin\varphi_2 \\ -l_2.\cos\varphi_2 \\ l_2.\sin\varphi_2 \\ -l_2.\cos\varphi_2 \end{bmatrix} \dot\varphi_2 \tag{5.44}$$

$$\begin{bmatrix} -l_3\sin\varphi_3 & -l_4\sin\varphi_4 & 0 & 0 \\ l_3\cos\varphi_3 & l_4\cos\varphi_4 & 0 & 0 \\ -l_3\sin\varphi_3 & 0 & -l_5\sin\varphi_5 & 0 \\ l_3\cos\varphi_3 & 0 & l_5\cos\varphi_5 & 1 \end{bmatrix} . \begin{bmatrix} \begin{bmatrix} \ddot{\varphi}_3 \\ \ddot{\varphi}_4 \\ \ddot{\varphi}_5 \\ \ddot{l}_{6y} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} l_2.\cos\varphi_2 \\ l_2.\sin\varphi_2 \\ l_2.\cos\varphi_2 \\ l_2.\sin\varphi_2 \end{bmatrix} \dot{\varphi}_2^2 + \begin{bmatrix} l_2.\sin\varphi_2 \\ -l_2.\cos\varphi_2 \\ l_2.\sin\varphi_2 \\ -l_2.\cos\varphi_2 \end{bmatrix} \ddot{\varphi}_2 + \tag{5.45}$$

$$+ \begin{bmatrix} l_3.\cos\varphi_3 \\ l_3.\sin\varphi_3 \\ l_3.\cos\varphi_3 \\ l_3.\sin\varphi_3 \end{bmatrix} \dot{\varphi}_3^2 + \begin{bmatrix} l_4.\dot{\varphi}_4^2\cos\varphi_4 \\ l_4.\dot{\varphi}_4^2\sin\varphi_4 \\ l_5.\dot{\varphi}_5^2\cos\varphi_5 \\ l_5.\dot{\varphi}_5^2\sin\varphi_5 \end{bmatrix}$$

$$\begin{aligned} \mathbf{A_1}.\mathbf{X_1} &= \mathbf{P}_1 \\ \mathbf{A_2}.\mathbf{X_2} &= \mathbf{P_2} \end{aligned} \tag{5.46}$$

where

$$\mathbf{A}_1 = \begin{bmatrix} -l_3\sin\varphi_3 & -l_4\sin\varphi_4 \\ l_3\cos\varphi_3 & l_4\cos\varphi_4 \end{bmatrix}, \quad \mathbf{X}_1 = \begin{bmatrix} \dot{\varphi}_3 \\ \dot{\varphi}_4 \end{bmatrix}, \quad \mathbf{P}_1 = \begin{bmatrix} l_2\sin\varphi_2 \\ -l_2\cos\varphi_2 \end{bmatrix} \dot{\varphi}_2,$$

$$\mathbf{A}_2 = \begin{bmatrix} -l_5\sin\varphi_5 & 0 \\ l_5\cos\varphi_5 & 1 \end{bmatrix}, \quad \mathbf{X}_2 = \begin{bmatrix} \dot{\varphi}_5 \\ \dot{l}_6 \end{bmatrix}, \quad \mathbf{P_2} = \begin{bmatrix} l_2\sin\varphi_2 \\ -l_2\cos\varphi_2 \end{bmatrix} \dot{\varphi}_2 + \begin{bmatrix} l_3\sin\varphi_3 \\ -l_3\cos\varphi_3 \end{bmatrix} \dot{\varphi}_3$$

$$\begin{aligned} \mathbf{A_1}.\dot{\mathbf{X}}_\mathbf{1} &= \dot{\mathbf{P}}_\mathbf{1} - \dot{\mathbf{A}}_\mathbf{1}.\mathbf{X}_1 \\ \mathbf{A_2}.\dot{\mathbf{X}}_\mathbf{2} &= \dot{\mathbf{P}}_\mathbf{2} - \dot{\mathbf{A}}_\mathbf{2}.\mathbf{X_2} \end{aligned} \tag{5.47}$$

where

$$\dot{\mathbf{P}}_1 = \begin{bmatrix} l_2\cos\varphi_2 \\ l_2\sin\varphi_2 \end{bmatrix} \dot{\varphi}_2^2 + \begin{bmatrix} l_2\sin\varphi_2 \\ -l_2\cos\varphi_2 \end{bmatrix} \ddot{\varphi}_2,$$

$$-\dot{\mathbf{A}}_\mathbf{1}\mathbf{X_1} = \begin{bmatrix} -l_3\cos\varphi_3 & -l_4\cos\varphi_4 \\ -l_3\sin\varphi_3 & -l_4\sin\varphi_4 \end{bmatrix} \begin{bmatrix} \dot{\varphi}_3{}^2 \\ \dot{\varphi}_4{}^2 \end{bmatrix} = \begin{bmatrix} l_3\cos\varphi_3\dot{\varphi}_3{}^2 & +l_4\cos\varphi_4\dot{\varphi}_4{}^2 \\ l_3\sin\varphi_3\dot{\varphi}_3{}^2 & +l_4\sin\varphi_4\dot{\varphi}_4{}^2 \end{bmatrix},$$

$$\dot{\mathbf{P}}_2 = \begin{bmatrix} l_2\cos\varphi_2 \\ -l_2\sin\varphi_2 \end{bmatrix} \dot{\varphi}_2{}^2 + \begin{bmatrix} l_2\sin\varphi_2 \\ -l_2\cos\varphi_2 \end{bmatrix} \ddot{\varphi}_2 + \begin{bmatrix} l_3\cos\varphi_3 \\ -l_3\sin\varphi_3 \end{bmatrix} \dot{\varphi}_3{}^2 + \begin{bmatrix} l_3\sin\varphi_3 \\ -l_3\cos\varphi_3 \end{bmatrix} \ddot{\varphi}_3,$$

$$-\dot{\mathbf{A}}_2\mathbf{X}_2 = \begin{bmatrix} -l_5\cos\varphi_5 & 0 \\ -l_5\sin\varphi_5 & 0 \end{bmatrix} \begin{bmatrix} \dot{\varphi}_5{}^2 \\ \dot{l}_6 \end{bmatrix} = \begin{bmatrix} l_5\cos\varphi_5 \\ l_5\sin\varphi_5 \end{bmatrix} \dot{\varphi}_5{}^2 .$$

It is important to point out that the following condition is valid:

$$\omega_{21} = \dot{\varphi}_2 = \frac{2\pi n}{60} = \frac{2\pi 3.5}{60} = 0{,}3665 rad/s, \quad \omega_{31} = \dot{\varphi}_3, \quad \omega_{41} = \dot{\varphi}_4, \quad \omega_{51} = \dot{\varphi}_5,$$

$$\mathbf{v}_{61} = -\dot{l}_{6y}$$

$$\alpha_{21} = \ddot{\varphi}_2 = 0 rad/s, \alpha_{31} = \ddot{\varphi}_3, \alpha_{41} = \ddot{\varphi}_4, \alpha_{51} = \ddot{\varphi}_5, \mathbf{a}_{61} = -\ddot{l}_{6y}$$

The results of the kinematic analysis:

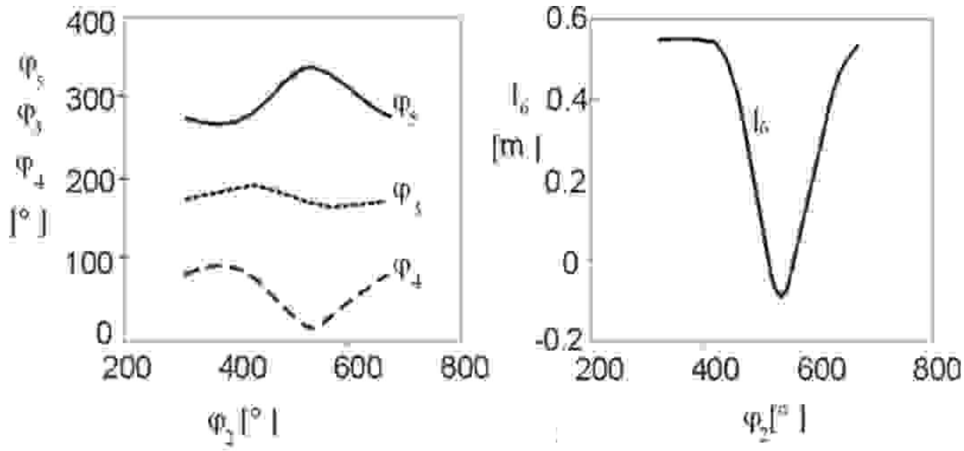


Fig. 5.38 Course of the position for individual items (bodies) of the mechanism

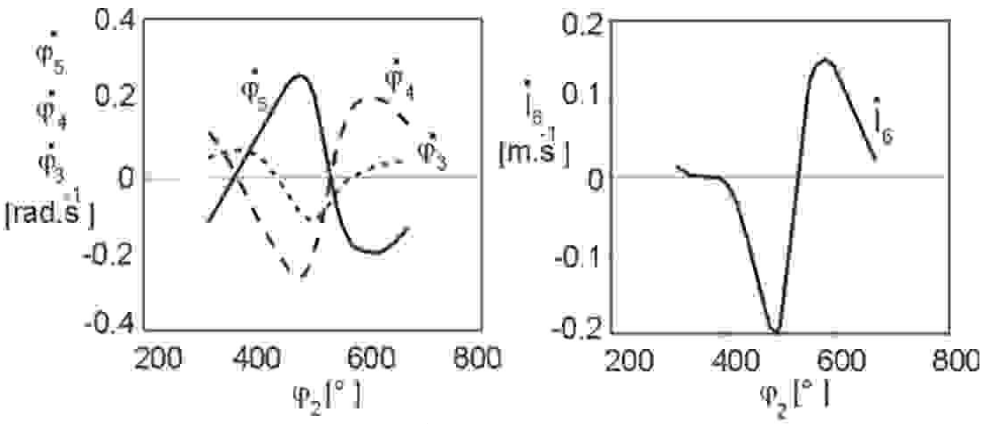


Fig. 5.39 Course of the speeds (velocities) for the individual bodies of the mechanism

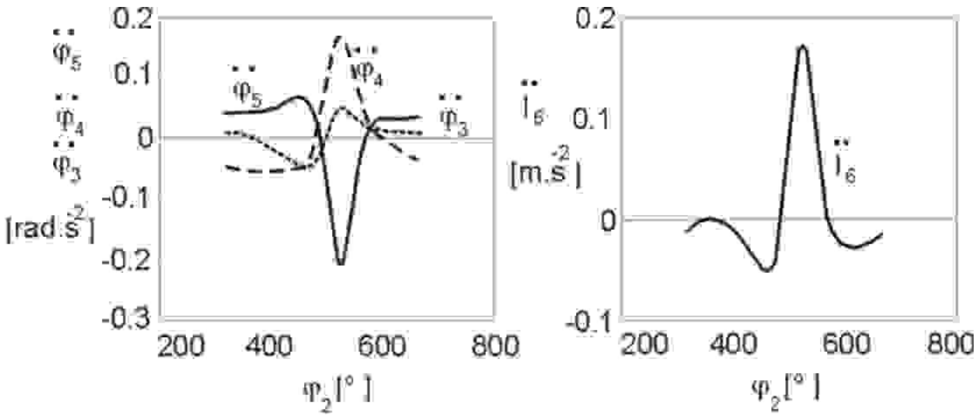


Fig. 5.40 Course of the acceleration for the individual bodies of the mechanism

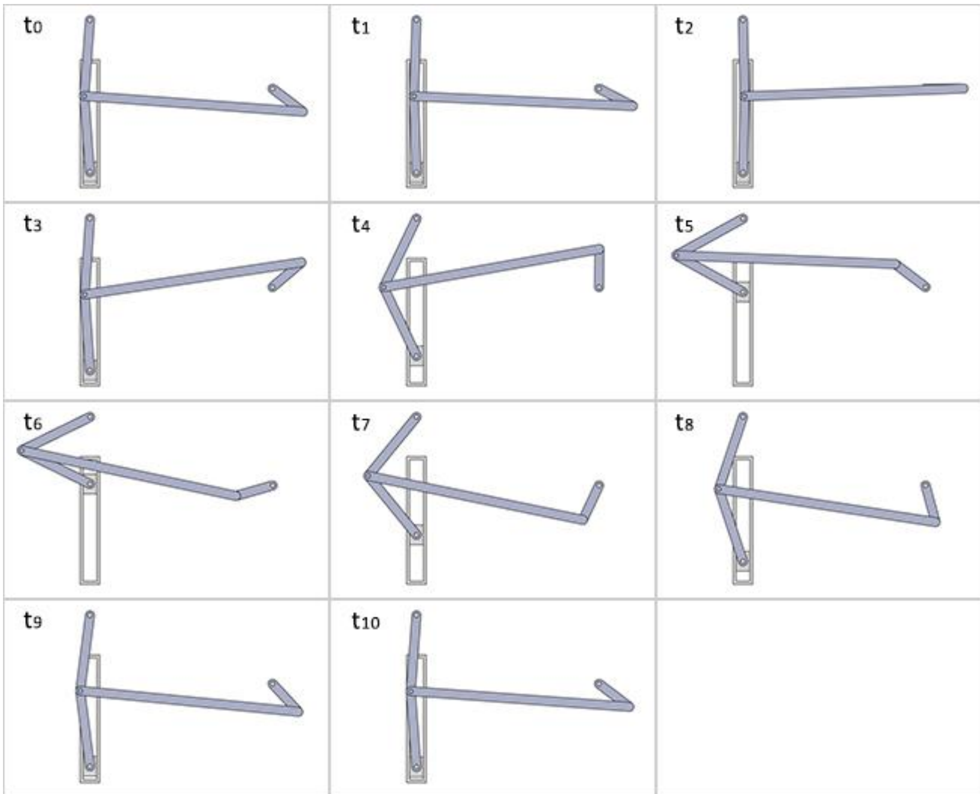


Fig. 5.41 Animation of the four positions of the mechanism

5.5.1 Dynamic analysis of planar mechanism

The main objective of the dynamic analysis is connected with specification of the loading for the individual items and determination of the courses relating to mutual reactions, referring to individual kinematic connections [4, 10–12]. The analysis was based on utilisation of the nonlinear model. Relating to the analysis, the other important values were utilised:

- modulus of elasticity (Young's modulus): $E = 210$ GPa,
- Poisson's ratio: $\mu = 0.3$,
- density of material: $\rho = 7850$ kg m^{-3}.

The analysis of the planar mechanism is based on selection of the linear tetrahedral element with four nodes (see chapter 2.2).

Fig. 5.42 represents the course of the reaction in D point of the body, designated as 6 and Fig. 5.43 represents the course of the reaction in A point of the body, designated as 3. Fig. 5.44 shows the degraded degrees of freedom of the given mechanism. The network of finite elements of the planar mechanism can be seen in Fig. 5.45.

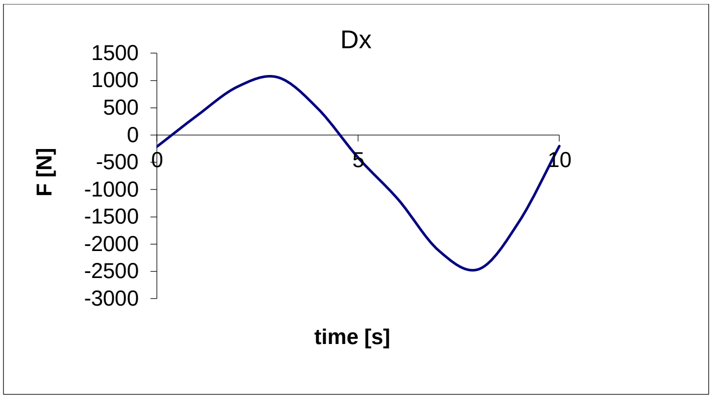


Fig. 5.42 Course of the reaction in D point of the item (body), designated as 6 – depending on time.

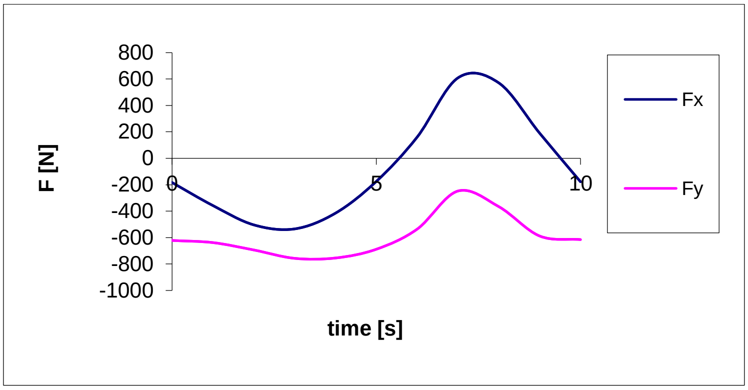


Fig. 5.43 Course of the reaction in A point of the item (body), designated as 3 – depending on time.

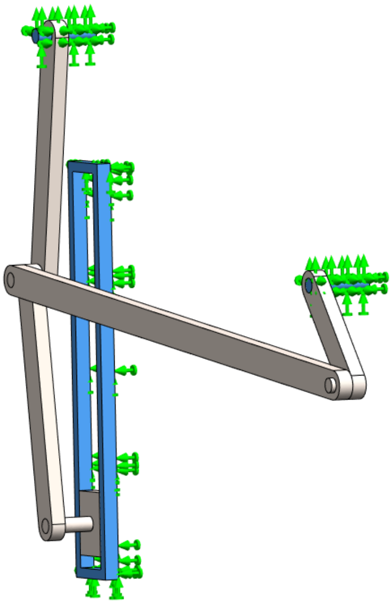


Fig. 5.44 Degraded degrees of freedom of the mechanism.

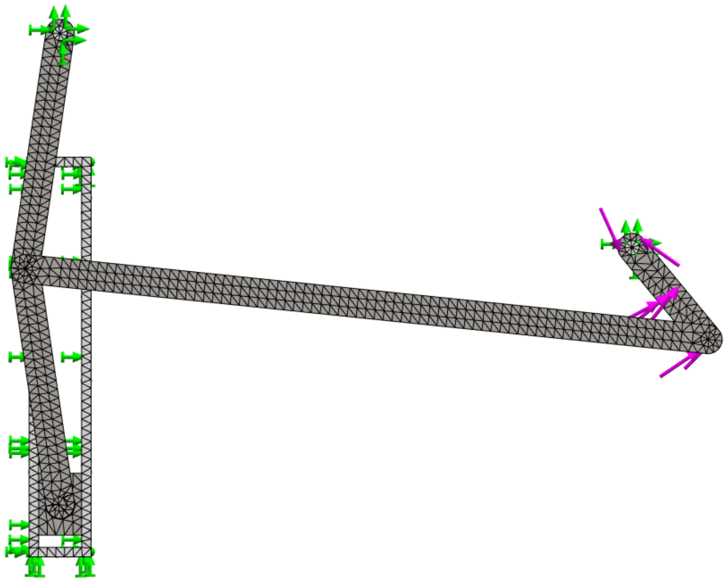


Fig. 5.45 Finite element network.

The distribution of the stress for mechanism [13, 14] can be seen in Fig. 5.46 and distribution of the displacement for mechanism is shown in Fig. 5.47.

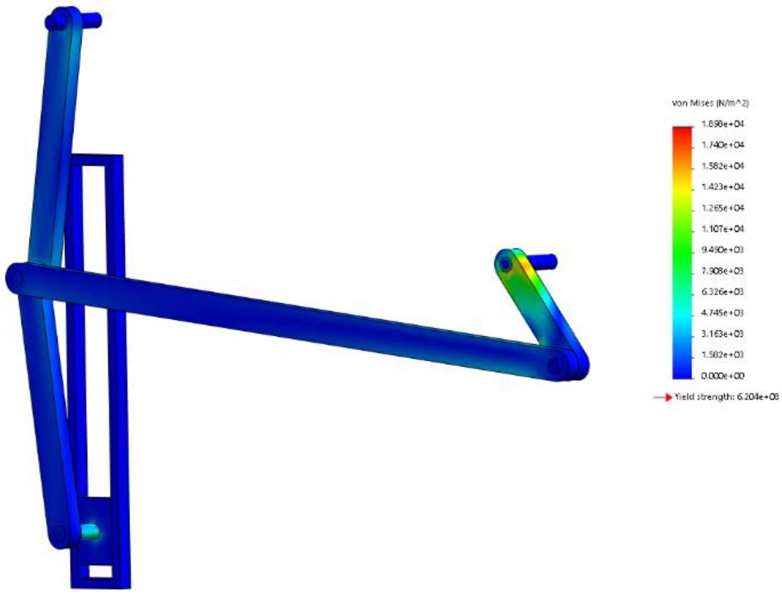


Fig. 5.46 Distribution of the stresses (Pa) for the mechanism.

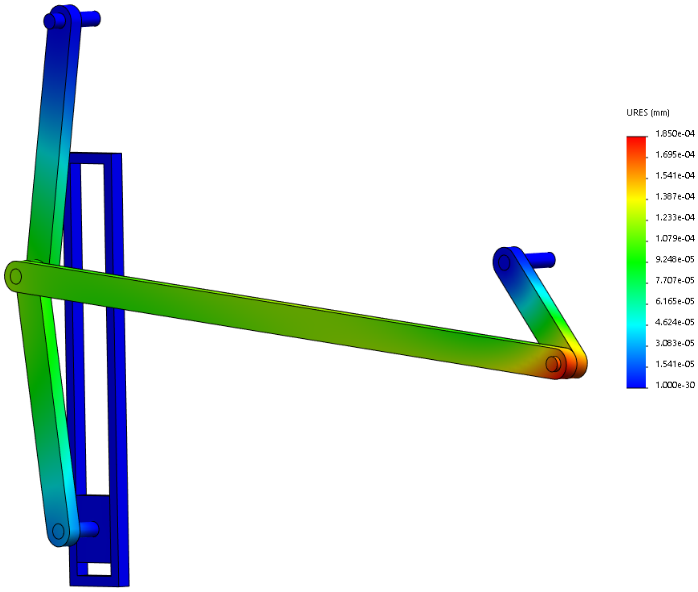


Fig. 5.47 Distribution of the displacement (mm) for the mechanism

5.6 Kinematic Analysis of the manipulator for removal of rough tyres

The manipulator for removal of rough tyres (Fig. 5.48) is composed of twenty-five individual bodies which are held together by help of kinematic connections and it is in the accordance with real state. The computational model of the manipulator can be seen in the Fig. 5.49.

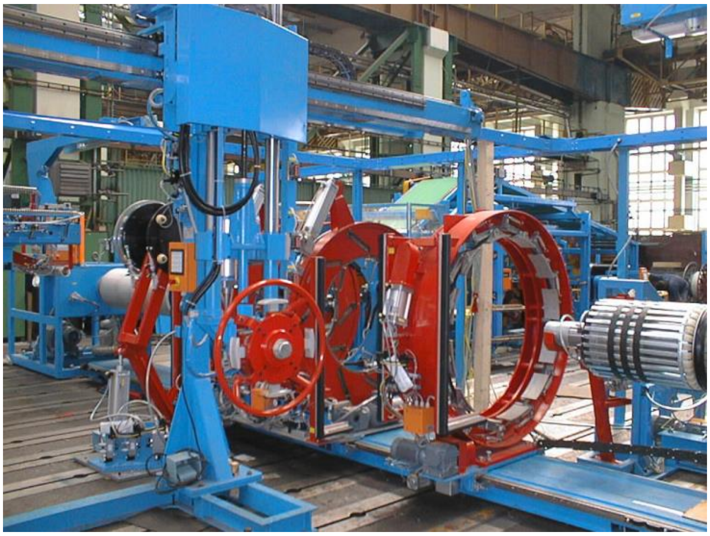


Fig. 5.48 The manipulator for removal of rough tyres

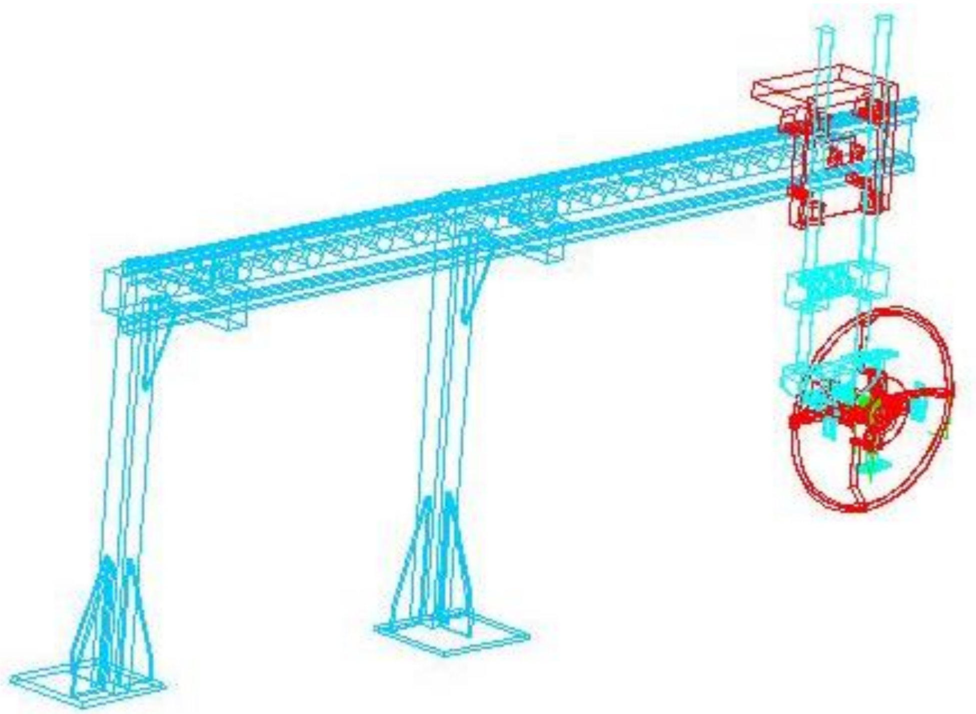


Fig. 5.49 Computational model of the manipulator

Using the kinematic analysis [32, 34, 35, 38], the main objective of solution is connected with the determination and entering of the position domains, speed (velocity) domains as well as acceleration of the individual items (members or bodies) in relation to the specified input values and it can be seen in the Tab. 5.1.

Tab. 5.1 Influences of external forces and kinematic phenomena on manipulator

1.	maximum gravity or load capacity, using 22.5” tyre	80 kg
2.	speed (velocity) of movement for manipulating item (member) in horizontal direction	400 $mm.s^{-1}$
3.	speed (velocity) of movement for manipulating item (member) in vertical direction	90 $mm.s^{-1}$
4.	speed (velocity) of disengaging for clamps used for removing	20 $mm.s^{-1}$

The simulation of operation relating to manipulator can be seen in the Fig. 5.50. In the given figure, there are 6 steps relating to technology of removing and manipulation with the tyre:

1. clamping of the tyre
2. removing of the tyre from the transferring of assembling or building up line
3. inspection of the tyre by operator
4. removing of the tyre above the rotating storage bin
5. tilting of the tyre
6. placing of the tyre into the rotating storage bin

It has to be pointed out that each one of the mentioned positions or steps is closely connected with specific influence referring to loading process of backbone frame of the manipulator.

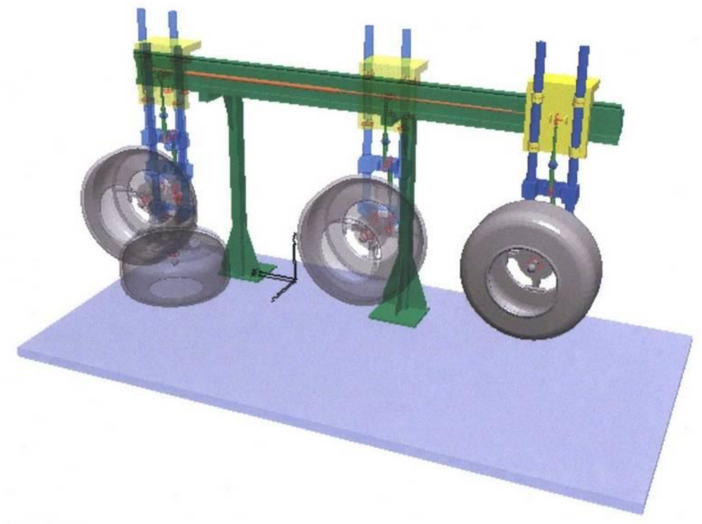


Fig. 5.50 Simulation of manipulator operation

In the Fig. 5.51, there is the tyre, which is clamped in the clamps of manipulator as well as there is also the physical model for clamping mechanism. The course of the speed (velocity) as well as the acceleration of the movement of displacement rail or frame can be seen in the Fig. 5.52 and Fig. 5.53.

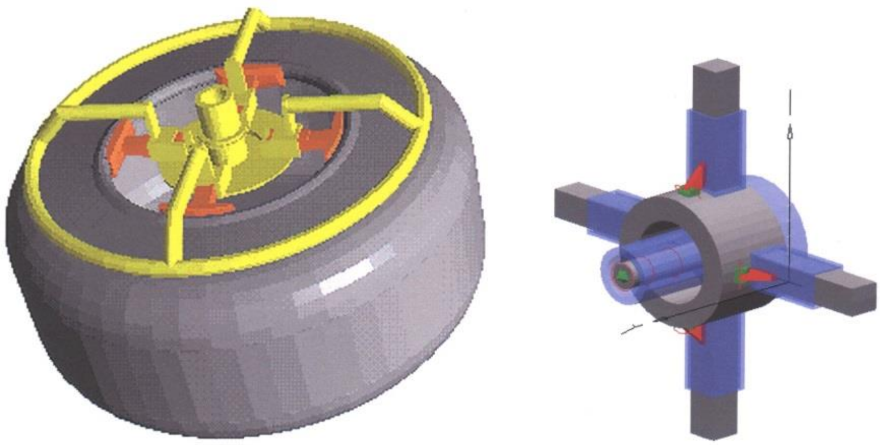


Fig. 5.51 Manipulator clamps and clamping mechanism for tyre clamping

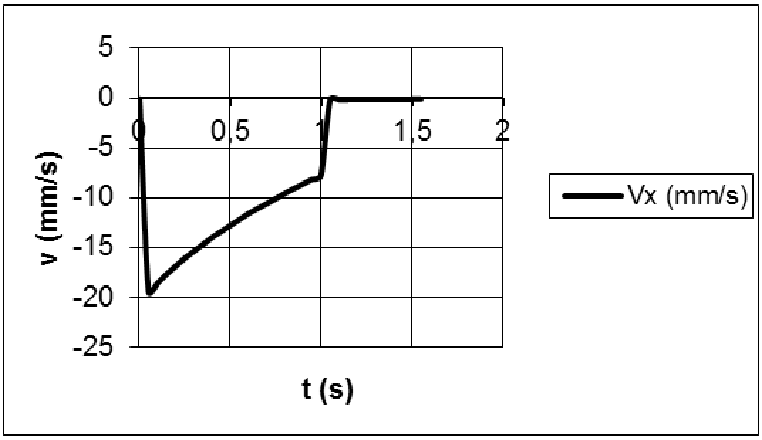


Fig. 5.52 Speed of the motion (movement) of the displacement rail (frame)

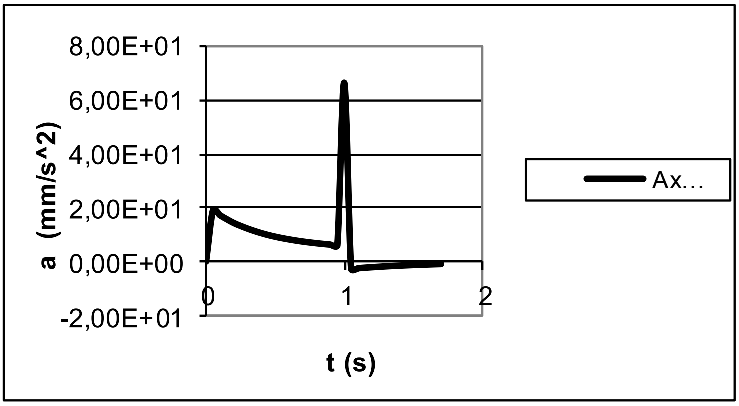


Fig. 5.53 Acceleration of the motion (movement) for the displacement rail (frame)

6 Kinematic and Dynamic Analysis and Distribution of Stress for Planar Mechanisms by Means of SolidWorks Software

In order to create a model of the mechanism, the calculation program uses the elements by help of which the resulting mechanism (system of bodies) is assembled and the motion equations of the mechanism are created by combination of the equations for the individual members or items of the mechanism. The individual elements are: connection of a body point with a frame, connection of points of two bodies, rotary (articulated or joint) connection of two bodies, sliding connection of two bodies, connection of two bodies by help of a drawbar with articulated (joint) end, with two sliding ends and combination of joint with sliding end, rope connection , wheels - rolling, cams, connection of a point with a curve and a surface, spherical joints and their various connections, linear spring and damper, absolute drives, relative drives and linear drives.

6.1 Kinematic and Dynamic Analysis and Distribution of Stress for Four-Item Mechanism

The planar mechanism representative (Fig. 6.1) consists of four bodies and was used as computational model. Using the kinematic analysis and dynamic analysis and subsequent simulation [30], the main objective is connected with the determination and entering of the position domains, speed (velocity) domains as well as acceleration of the individual bodies in relation to the specified input values of the angular velocity for the driving body, designated as 2. The angular velocity for the body, designated as 2, is specified in this way: ω_{21}=36 [°/s], where ω_{21}=36 [°/s] is constant and it is changed in dependence on time (Fig. 6.2).
Specified input values can be seen in (Fig. 6.1).

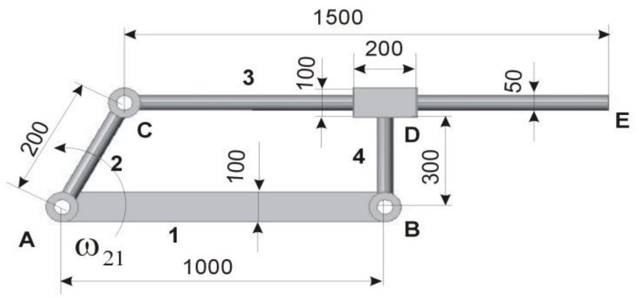


Fig. 6.1 Planar mechanism – computational model

Course of input value for angular velocity can be seen in Fig. 6.2 and angular acceleration of 2, 3, 4 bodies in dependence on time can be seen in Fig. 6.3.

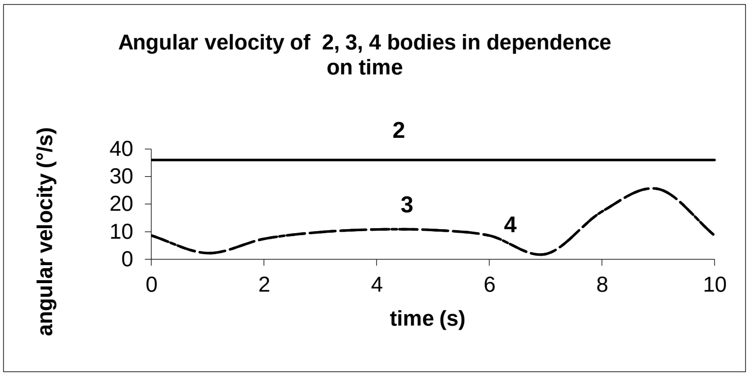


Fig. 6.2 Angular velocity of 2, 3, 4 bodies in dependence on time

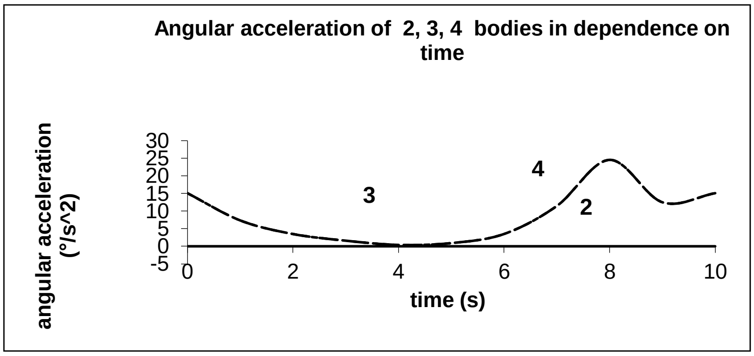


Fig. 6.3 Angular acceleration of 2, 3, 4 bodies in dependence on time

The simulation of operation relating to planar mechanism can be seen in the Fig. 6.4 and it is for time step referring to one second while the whole simulation takes place for ten seconds.

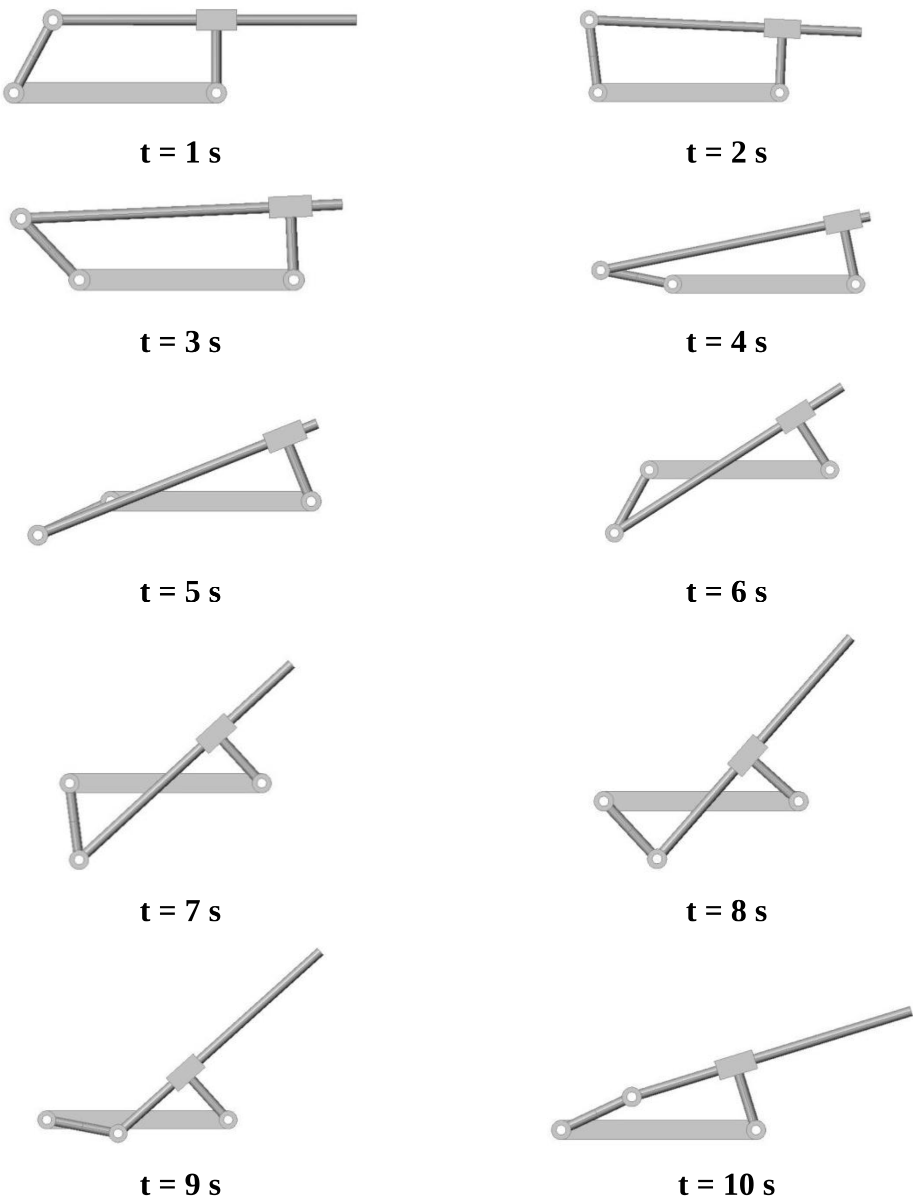


Fig. 6.4 Simulation of planar mechanism operation for ten positions

The whole course of the velocity and acceleration for C, D, E points of bodies can be seen in Fig.6.5 and Fig.6.6.

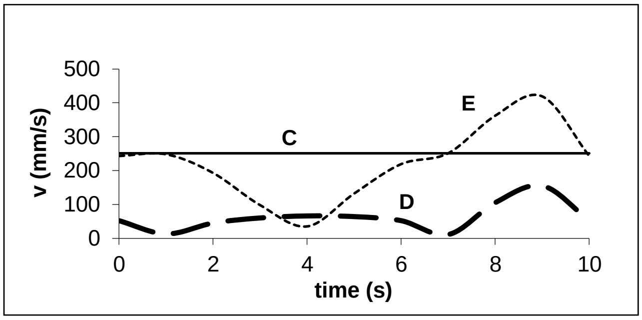


Fig. 6.5 Velocity in points (C, D, E) – in dependence on the time

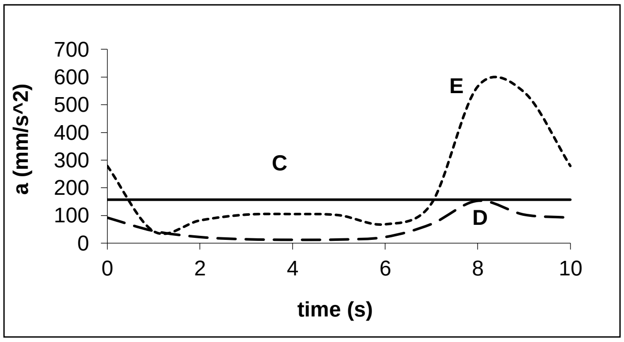


Fig. 6.6 Acceleration in points (C, D, E) – in dependence on the time

The main objective of the dynamic analysis is connected with specification of the loading for the individual items and determination of the courses relating to mutual reactions, referring to individual kinematic connections. Fig. 6.7 represents the course of the reaction in C point of the body, designated as 2 and Fig.6.8 represents the course of the reaction in B point of the body, designated as 4.

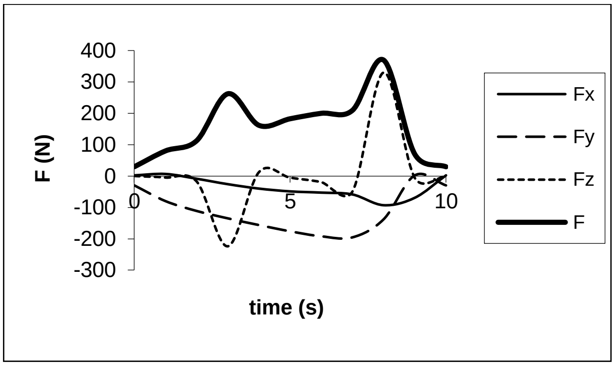


Fig. 6.7 Course of the reaction in C point of the body, designated as 2 – in dependence on the time

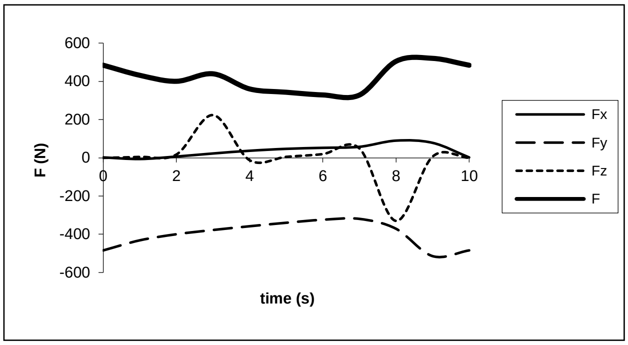


Fig. 6.8 Course of the reaction in B point of the body, designated as 4 –in dependence on the time

6.1.1 Type of finite elements and material properties

The analysis of the planar mechanism is based on selection of the linear tetrahedral element with four nodes (see chapter 2.2).

The analysis is based on utilisation of the linear model. Relating to the analysis, the other important values are utilised:

- modulus of elasticity (Young's modulus): $E = 2.1\ e^{11}$ (Pa),

- Poisson's ratio: $\mu = 0.3$,

- density of material: $\rho = 7850$ (kg.m^{-3}).

6.1.2 Distribution of the Stress in Items of Planar Mechanism

The distribution of the stress for connected bodies, designated as 1, 2, 3, 4, can be seen in Figs. 6.9-6.16.

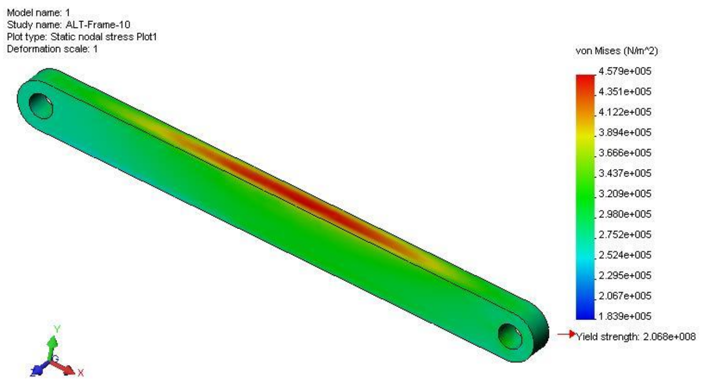


Fig. 6.9 Distribution of the stress for body, designated as 1 in [Pa]

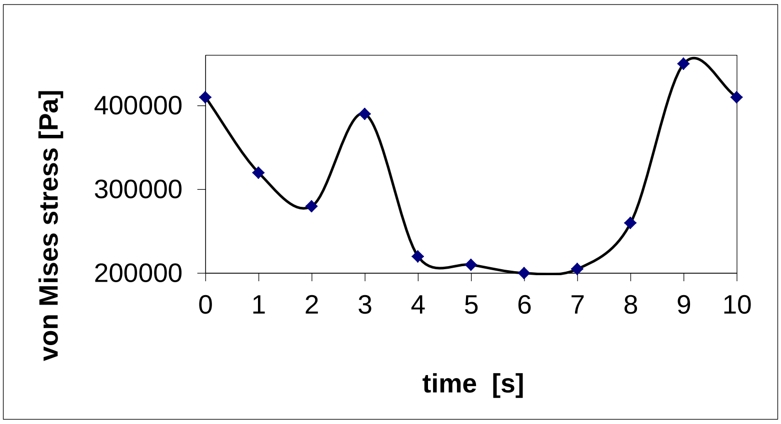


Fig. 6.10 Course of the stress for body, designated as 1 – in dependence on the time

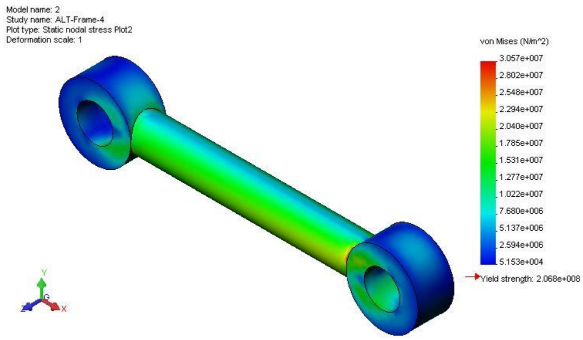


Fig. 6.11 Distribution of the stress for body, designated as 2 in [Pa]

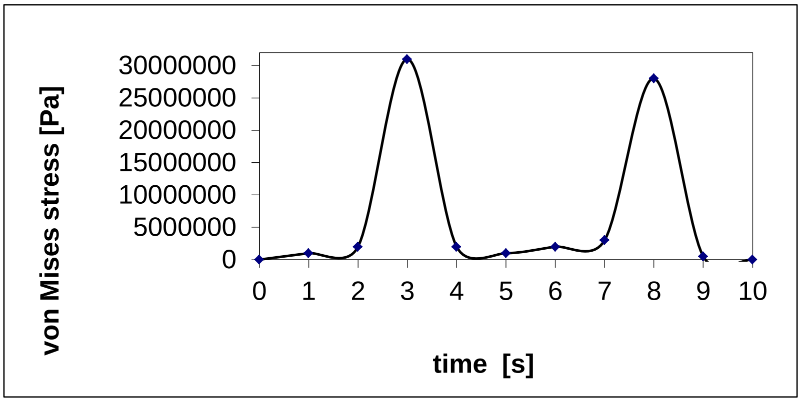


Fig. 6.12 Course of the stress for body, designated as 2 – in dependence on the time

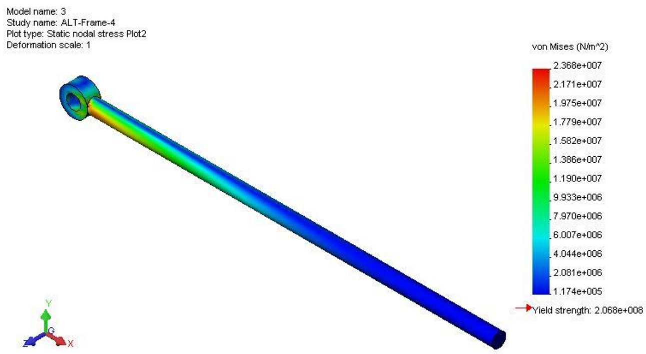


Fig. 6.13 Distribution of the stress for body, designated as 3 in [Pa]

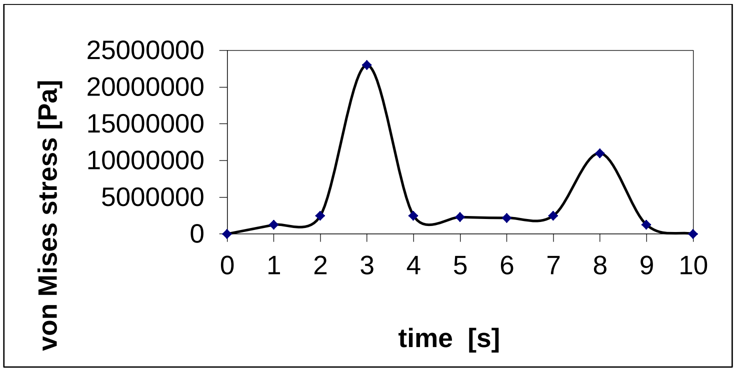


Fig. 6.14 Course of the stress for body, designated as 3 – in dependence on the time

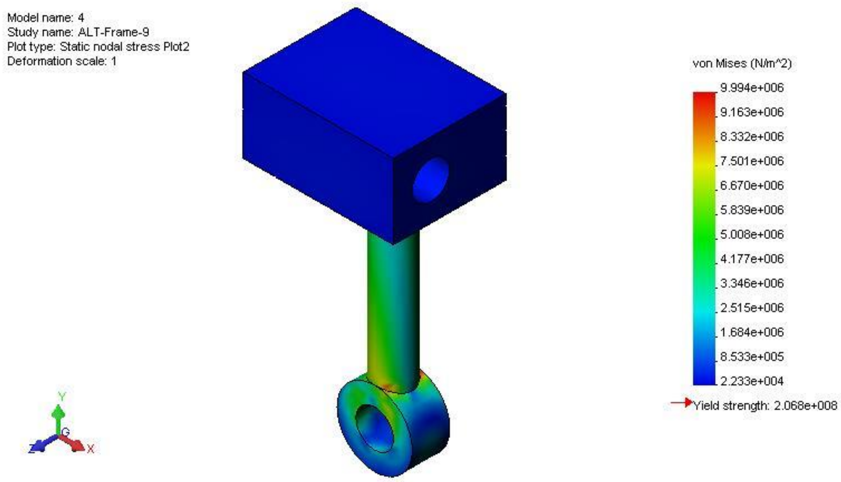


Fig. 6.15 Distribution of the stress for body, designated as 4 in [Pa]

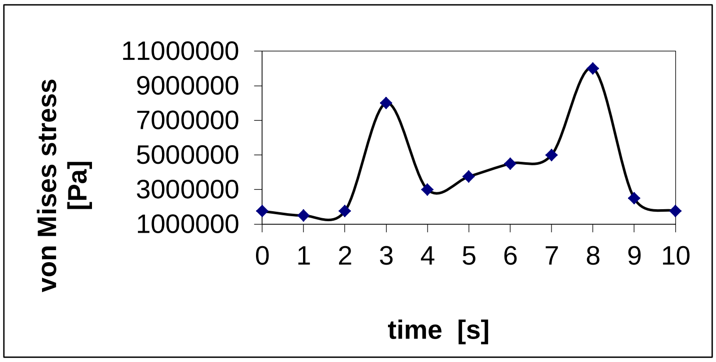


Fig. 6.16 Course of the stress for body, designated as 4 – in dependence on the time

6.2 Kinematic Analysis and Distribution of Stress for Five-Item Mechanism

The planar mechanism representative (Figure 6.17) consists of five bodies and it was used as computational model. Using the kinematic analysis and subsequent simulation, the main objective is connected with the determination and entering of the position domains, speed (velocity) domains as well as acceleration of the individual bodies in relation to the specified input values of the angular velocity for the driving body designated as 2. The angular velocity for the body, designated as 2, is specified in this way: ω21=36 [°/s], where ω21=36 [°/s] is constant (Figure 6.23). The dimensional parameters of individual bodies are shown in Figures 6.18-6.22.

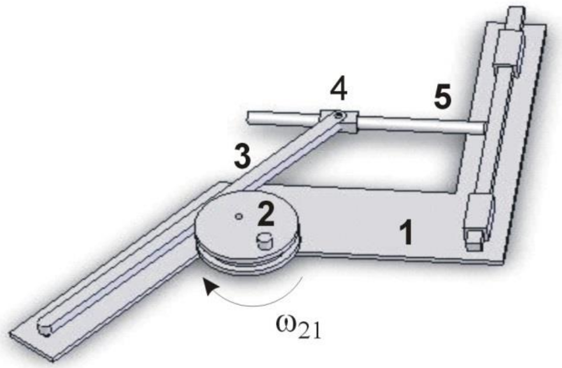


Fig. 6.17 Planar mechanism – computational model

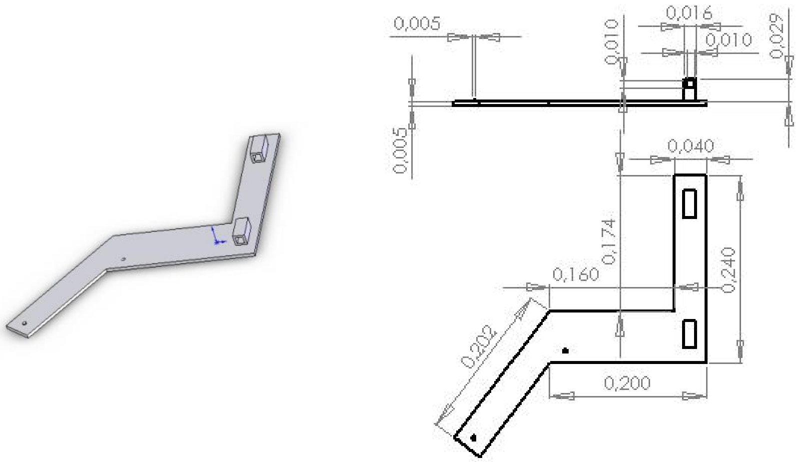


Fig. 6.18 Frame, designated as 1 and its dimensional parameters in [m]

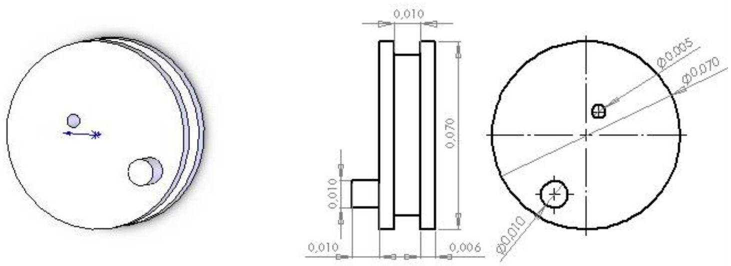


Fig. 6.19 Item or body (member), designated as 2 and its dimensional parameters in [m]

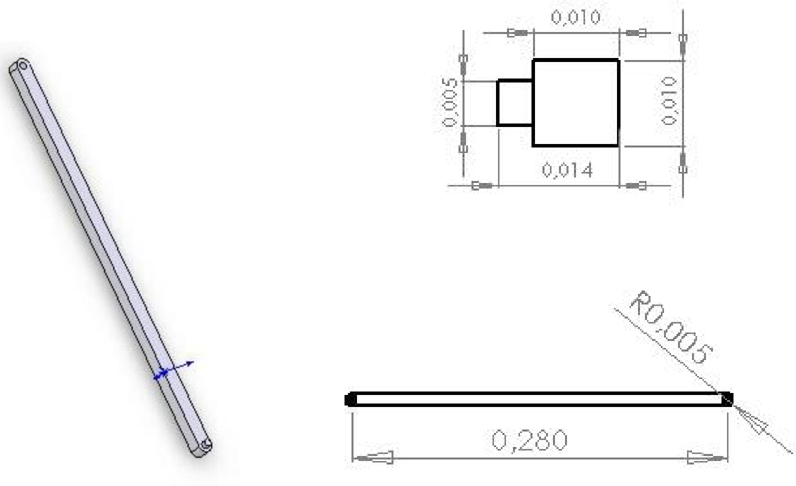


Fig. 6.20 Item or body (member) 3 and its dimensional parameters in [m]

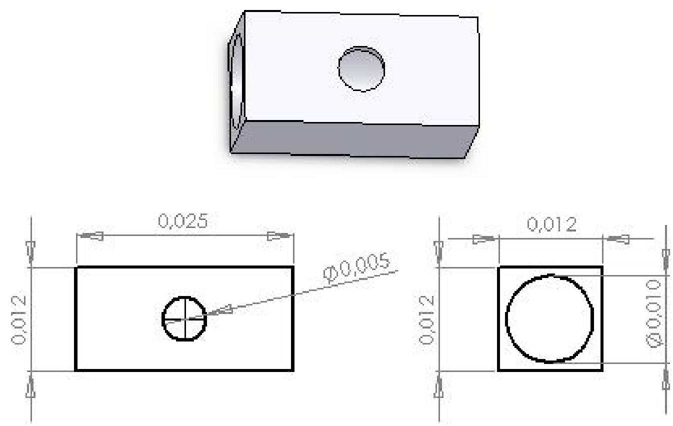


Fig. 6.21 Item 4 and its dimensional parameters in [m]

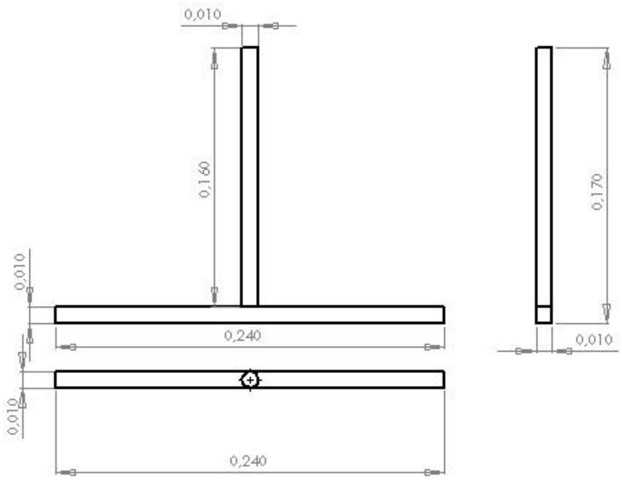


Fig. 6.22 Item or body (member) 5 and its dimensional parameters in [m]

Course of input value for angular velocity can be seen in Figure 6.23 and angular acceleration of 2, 3, 4, 5 bodies in dependence on time can be seen in Figure 6.24.

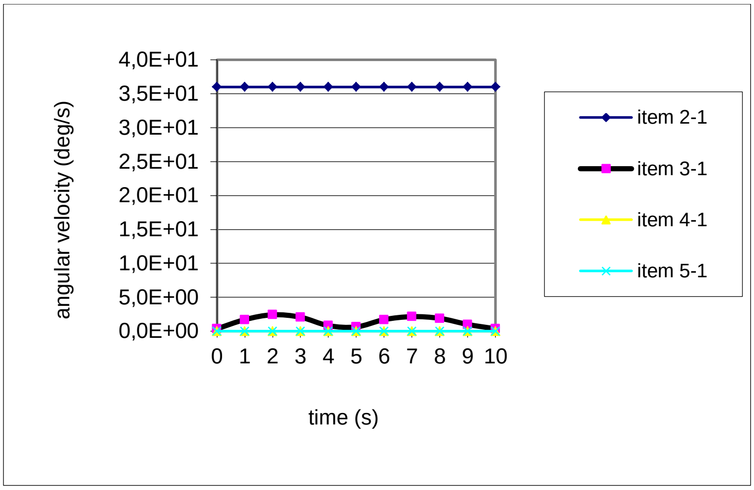


Fig. 6.23 Angular velocity of 2, 3, 4, 5 bodies (members or items) in dependence on the time

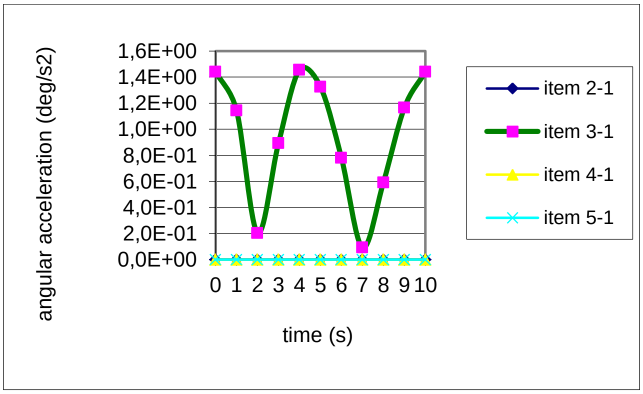
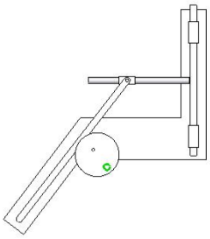
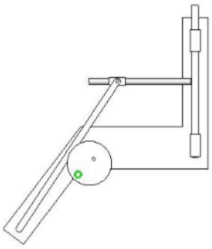


Fig. 6.24 Angular acceleration of 2, 3, 4, 5 bodies (members or items) in dependence on the time

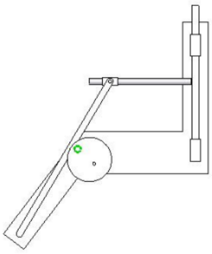
The simulation of 4 positions can be seen in Figure 6.25.



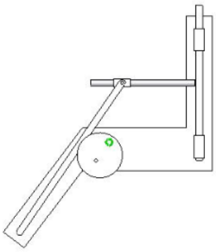
time = 0 s



time = 1 s



time = 7 s



time = 8 s

Fig. 6.25 Simulation of planar mechanism operation for four positions

Course of the velocity or speed can be seen in Figure 6.26 and acceleration of 2, 3, 4, 5 bodies in dependence on time can be seen in Figure 6.27.

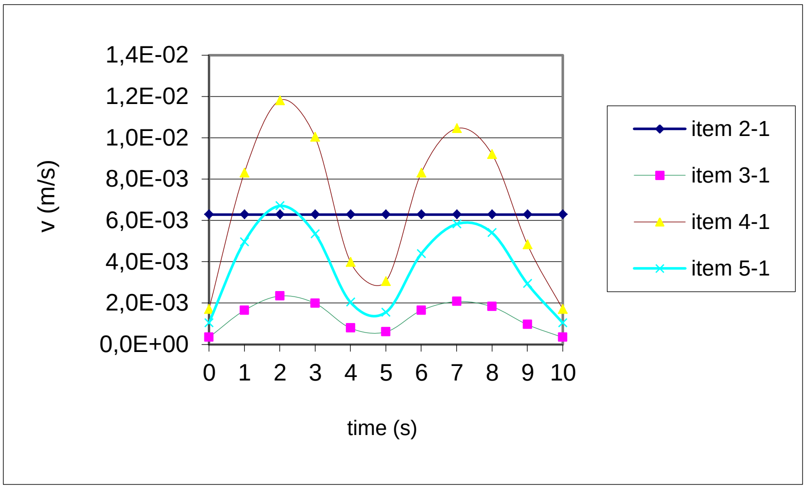


Fig. 6.26 Velocity or speed of bodies for 2, 3, 4, 5 – in dependence on the time

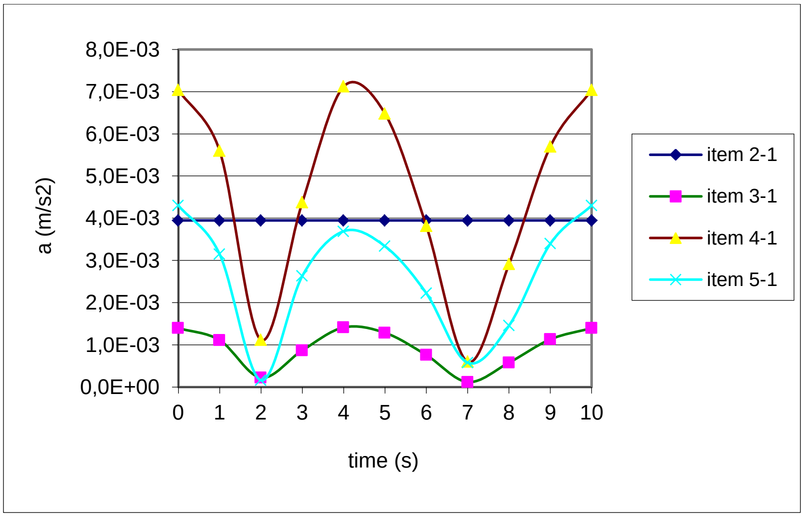


Fig. 6.27 Acceleration of bodies for 2, 3, 4, 5 – in dependence on the time

6.2.1 Type of finite elements and material properties

The analysis of the planar mechanism is based on selection of the linear tetrahedral element with four nodes (see chapter 2.2).

The analysis is based on utilisation of the linear model. Relating to the analysis, the other important values are utilised:

- modulus of elasticity (Young's modulus): $E = 2.1\ e^{11}$ (Pa),
- Poisson's ratio: $\mu = 0.3$,
- density of material: $\rho = 7850$ (kg.m^{-3}).

6.2.2 Distribution of the Stress in Items of Planar Mechanism

The distribution of the stress for individual bodies (items or members), designated as 1, 2, 3, 5, can be seen in Figs. from 6.28 to 6.35.

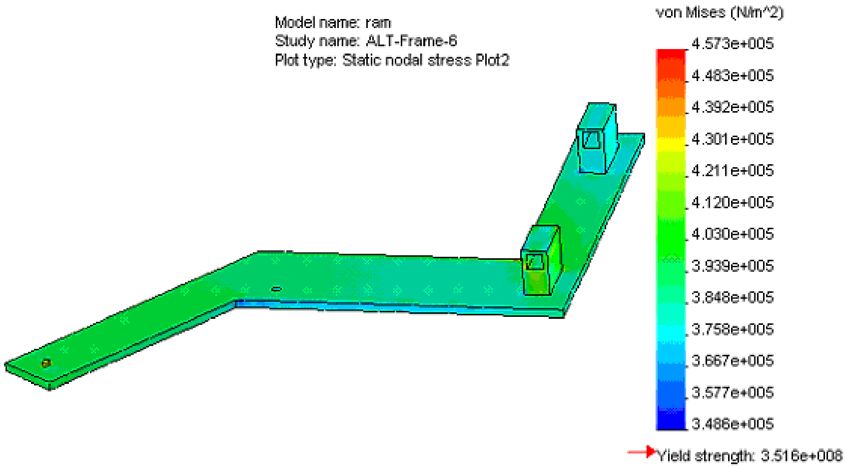


Fig. 6.28 Distribution of the stresses [Pa] for body, designated as 1

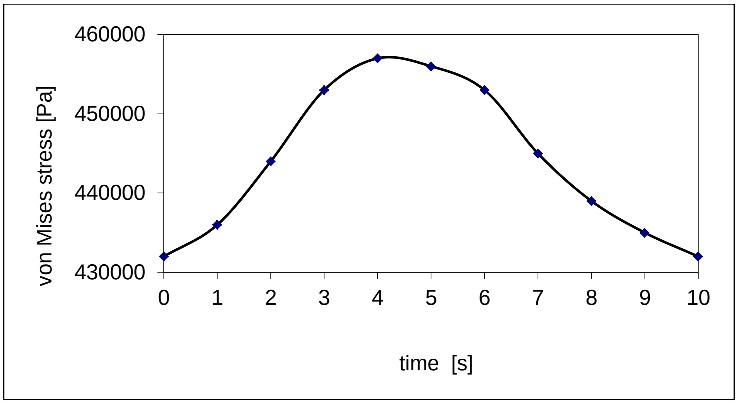


Fig. 6.29 Course of the stresses for body, designated as 1 – in dependence on the time

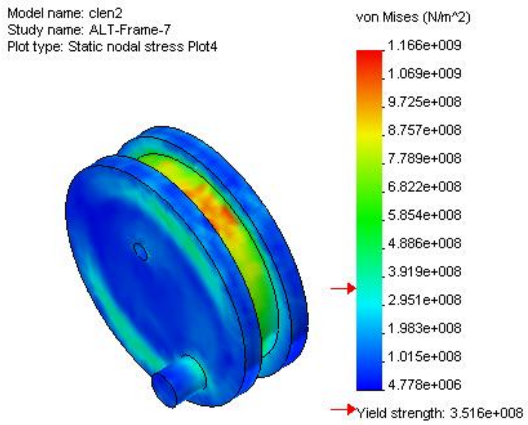


Fig. 6.30 Distribution of the stresses [Pa] for body, designated as 2

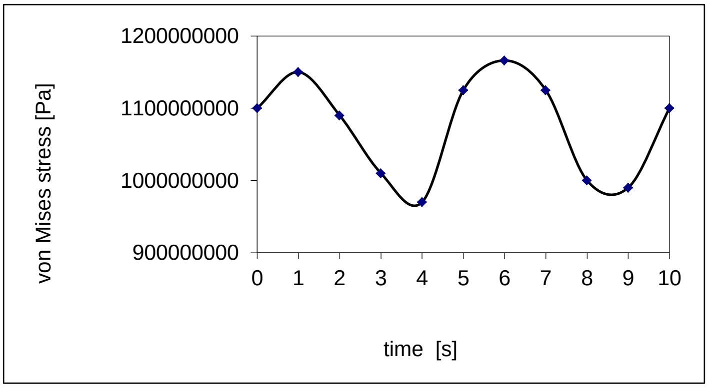


Fig. 6.31 Course of the stresses for body, designated as 2 – in dependence on the time

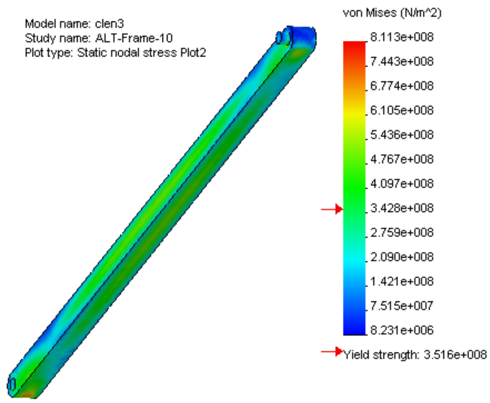


Fig. 6.32 Distribution of the stresses [Pa] for body, designated as 3

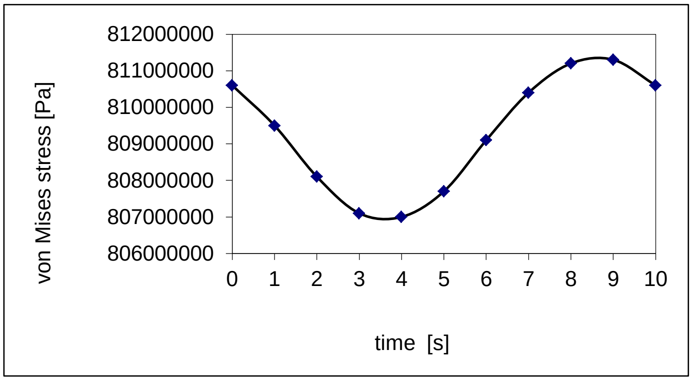


Fig. 6.33 Course of the stresses for body, designated as 3 – in dependence on the time

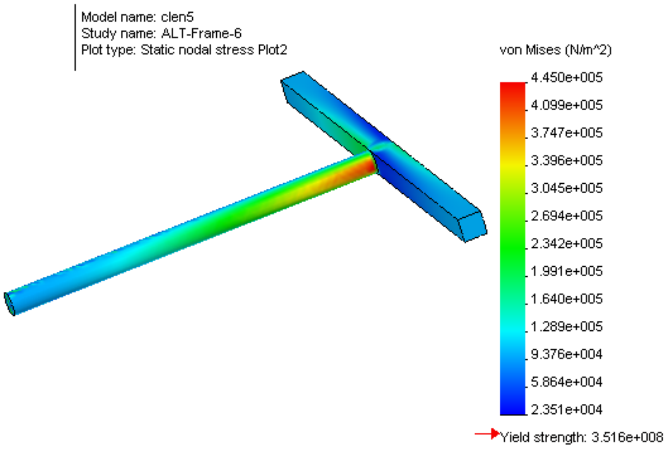


Fig. 6.34 Distribution of the stresses [Pa] for body, designated as 5

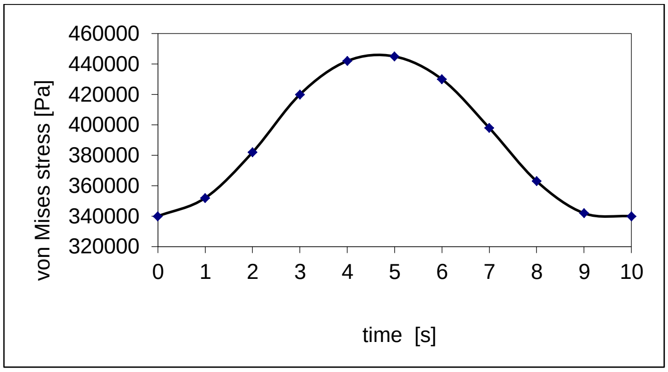


Fig. 6.35 Course of the stresses for body, designated as 5 – in dependence on the time

6.3 Kinematic and Dynamic Analysis and Distribution of Stress for Six-Item Mechanisms

The planar mechanism representative (Fig. 6.36) consists of six bodies and was used as computational model. Using the kinematic analysis and dynamic analysis and subsequent simulation [33], the main objective is connected with the determination and entering of the position domains, speed (velocity) domains as well as acceleration of the individual bodies in relation to the specified input values of the angular velocity for the driving body designated as 2. The angular velocity for the body, designated as 2, is specified in this way: ω_{21}=1 [°/s] and α_{21}=0,7 [°/s^2], where ω_{21}=1 [°/s] is not constant and it is changed in dependence on time (Fig. 6.37). Specified input values can be seen in Fig. 6.38.

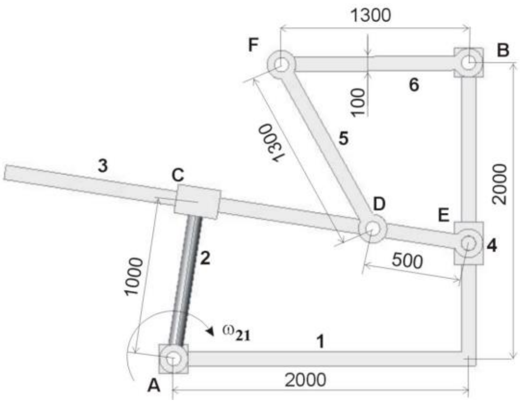


Fig. 6.36 Planar mechanism – computational model

Course of input value for angular velocity and angular acceleration is in Fig. 6.37 and Fig. 6.38.

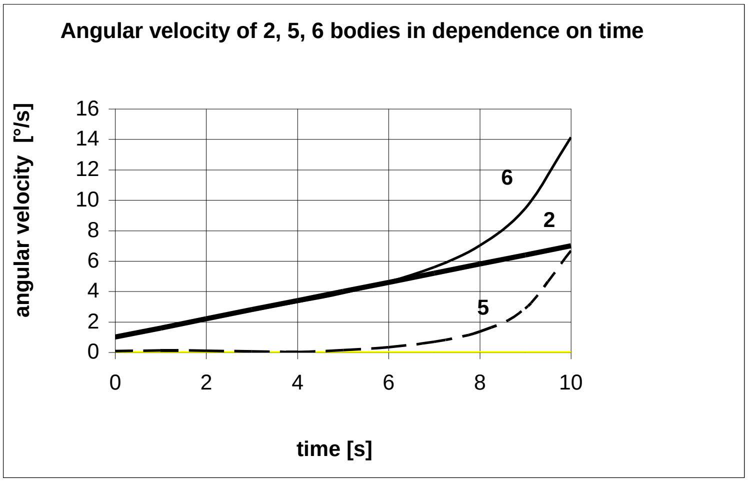


Fig. 6.37 Angular velocity of 2, 5, 6 bodies in dependence on the time

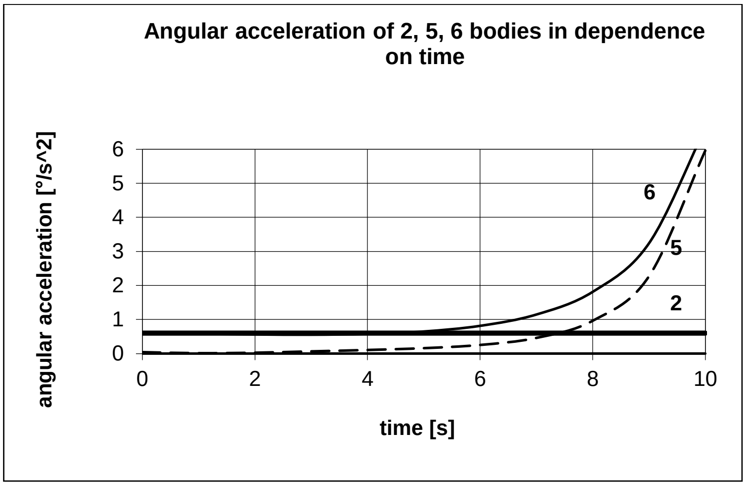


Fig. 6.38 Angular acceleration of 2, 5, 6 bodies in dependence on the time

The simulation of operation relating to planar mechanism can be seen in the Fig. 6.39 for time step referring to one second while the whole simulation takes place for ten seconds.

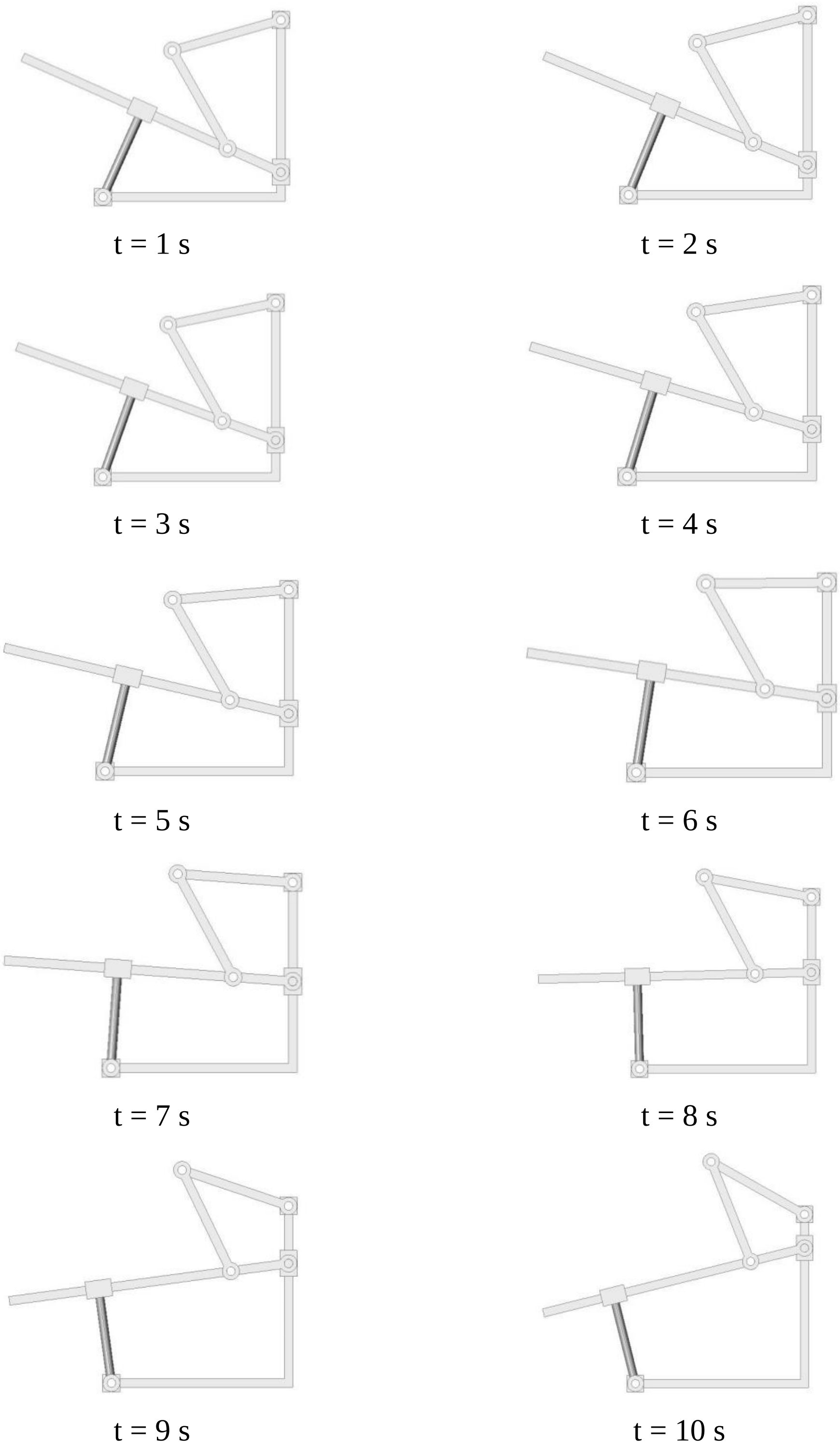


Fig. 6.39 Simulation of planar mechanism operation for ten positions

The whole course of the velocity and acceleration for C, D, E, F points of bodies can be seen in Fig.6.40 and Fig.6.41.

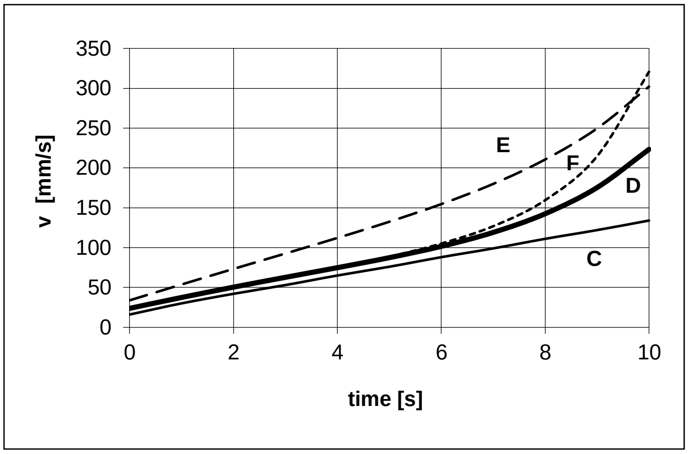


Fig. 6.40 Velocity in points (C, D, E, F) – in dependence on the time

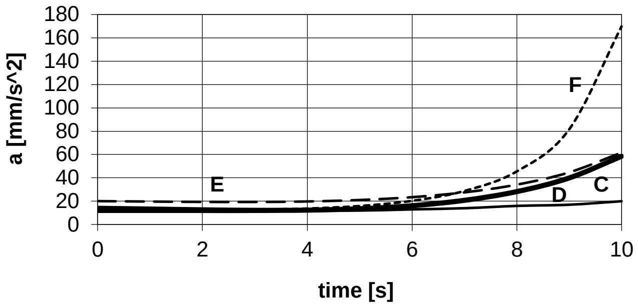


Fig. 6.41 Acceleration in points (C, D, E, F) – in dependence on the time

The main objective of the dynamic analysis is connected with specification of the loading for the individual items and determination of the courses relating to mutual reactions, referring to individual kinematic connections.

Fig. 6.42 represents the course of the reaction in F point of the body, designated as 6 and Fig.6.43 represents the course of the reaction in D point of the body, designated as 5.

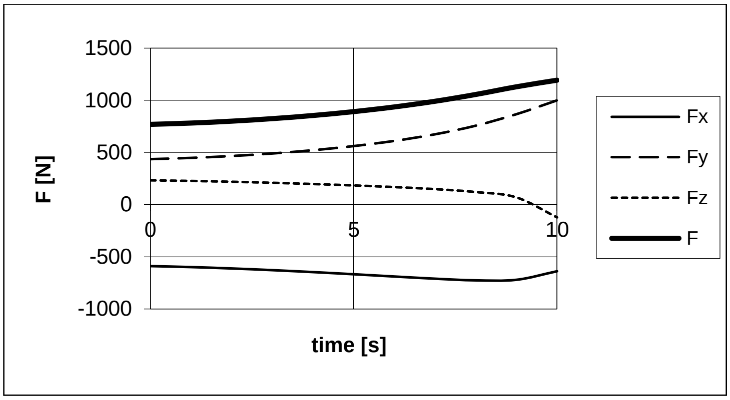


Fig. 6.42 Course of the reaction in F point of the body, designated as 6 – in dependence on the time

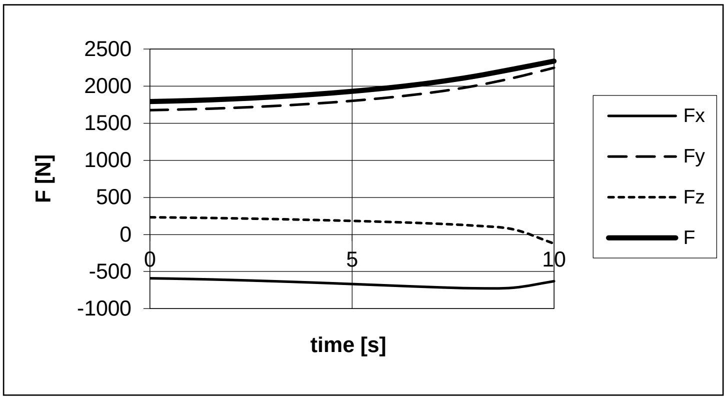


Fig. 6.43 Course of the reaction in D point of the body, designated as 5 – in dependence on the time

6.3.1 Type of finite elements and material properties

The analysis of the planar mechanism was based on selection of the linear tetrahedral element with four nodes (see chapter 2.2).

The analysis was based on utilisation of the linear model. Relating to the analysis, the other important values were utilised:

- modulus of elasticity (Young's modulus): $E = 2.1\ e^{11}$ (Pa),
- Poisson's ratio: $\mu = 0.3$,
- density of material: $\rho = 7850$ ($kg.m^{-3}$).

6.3.2 Distribution of the Stress in Items of Planar Mechanism

The distribution of the stress for connected bodies (items or members), designated as 1, 2, 3, 5 can be seen in Figs. from 6.44 to 6.51.

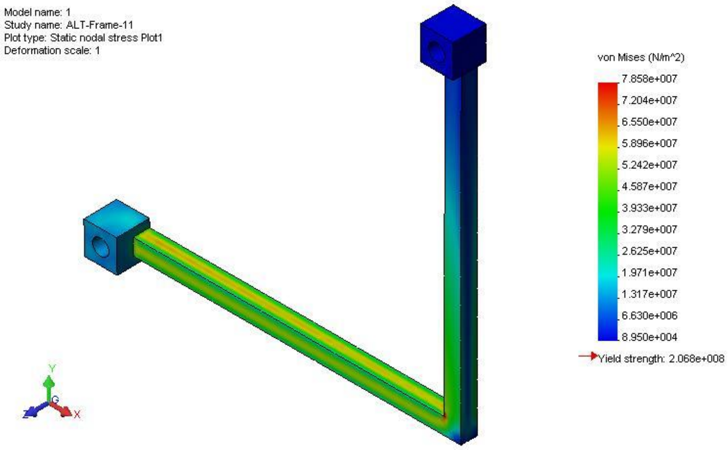


Fig. 6.44 Distribution of the stress for body, designated as 1 in [Pa]

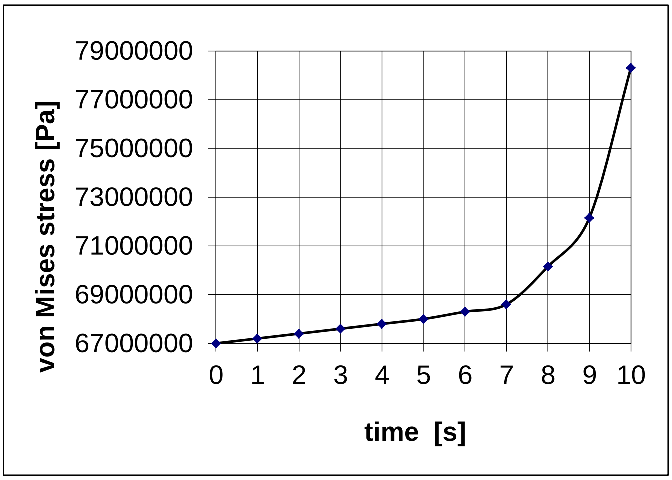


Fig. 6.45 Course of the stress for body, designated as 1 – in dependence on the time

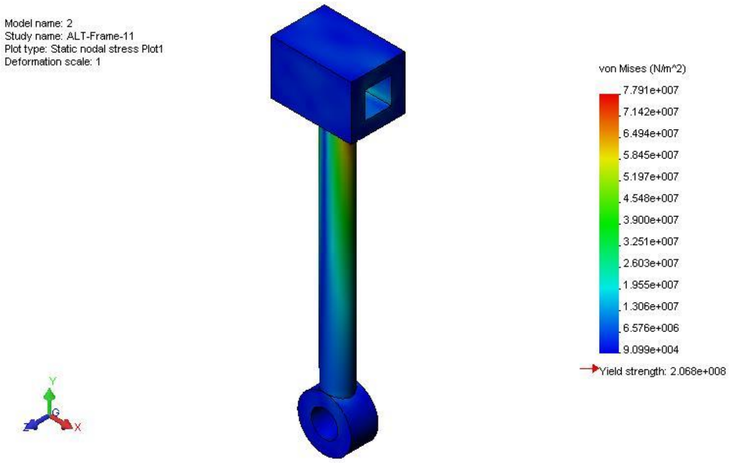


Fig. 6.46 Distribution of the stress for body, designated as 2 in [Pa]

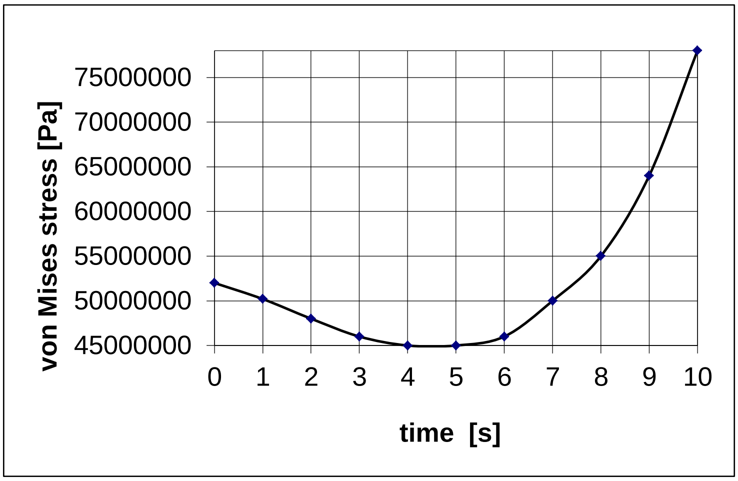


Fig. 6.47 Course of the stress for body, designated as 2 – in dependence on the time

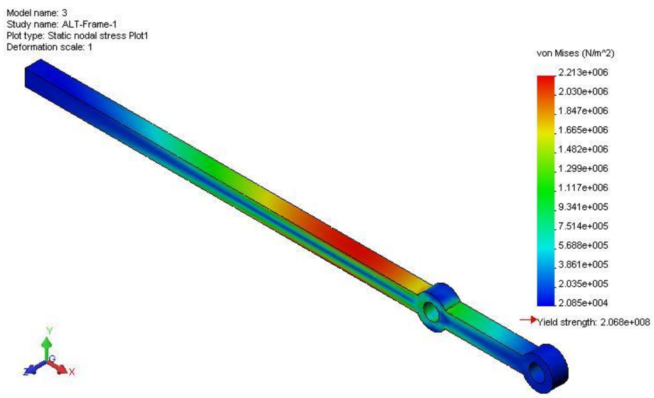


Fig. 6.48 Distribution of the stress for body, designated as 3 in [Pa]

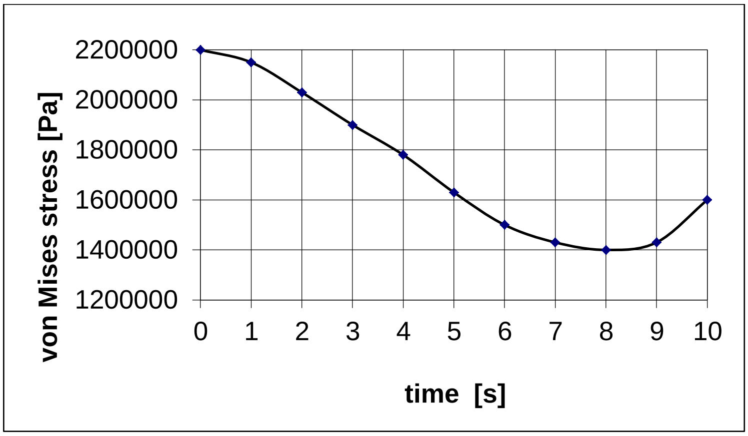


Fig. 6.49 Course of the stress for body, designated as 3 – in dependence on the time

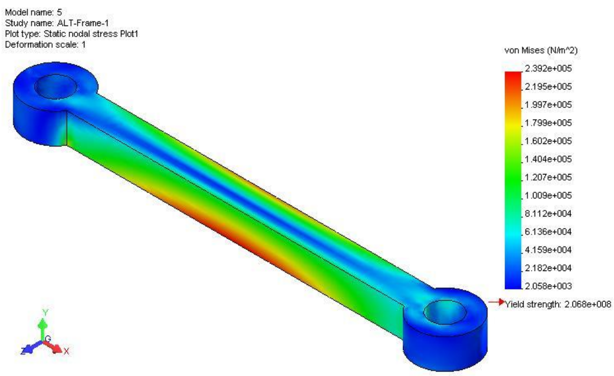


Fig. 6.50 Course of the stress for body, designated as 5 in [Pa]

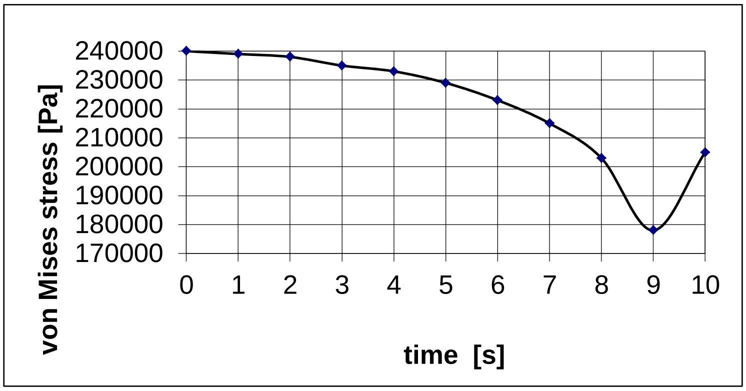


Fig. 6.51 Course of the stress for body, designated as 5 – in dependence on the time

Example 1

The planar mechanism representative (Fig. 6.52) consists of six bodies and it is used as computational model. Using the kinematic analysis and dynamic analysis and subsequent simulation [33], the main objective is connected with the determination and entering of the position domains, speed (velocity) domains as well as acceleration of the individual bodies in relation to the specified input values of the angular velocity for the driving body, designated as 2. The angular velocity for the body, designated as 2, is specified in this way: ω_{21}=20 [°/s] and α_{21}=0,7 [°/s^2], where ω_{21} is not constant and it is changed in dependence on the time (Fig.6.53). Specified input values can be seen in Figure 6.54.

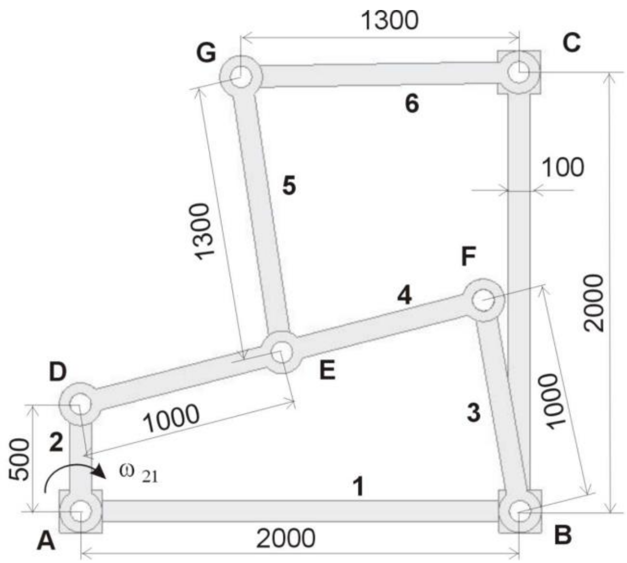


Fig. 6.52 Planar mechanism – computational model

Course of input value for angular velocity and angular acceleration is in Fig.6.53 and Fig.6.54.

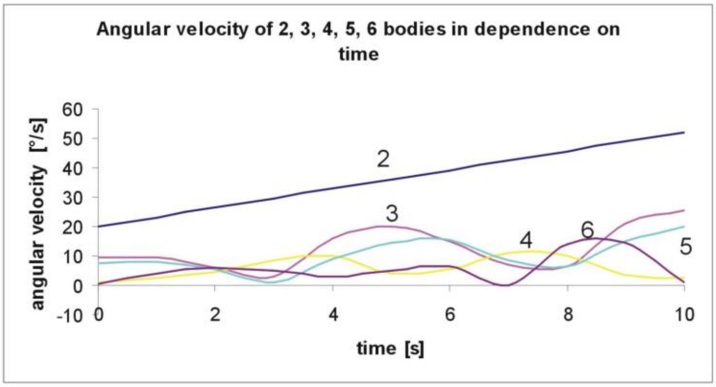


Fig. 6.53 Angular velocity of 2, 3, 4, 5, 6 bodies in dependence on the time

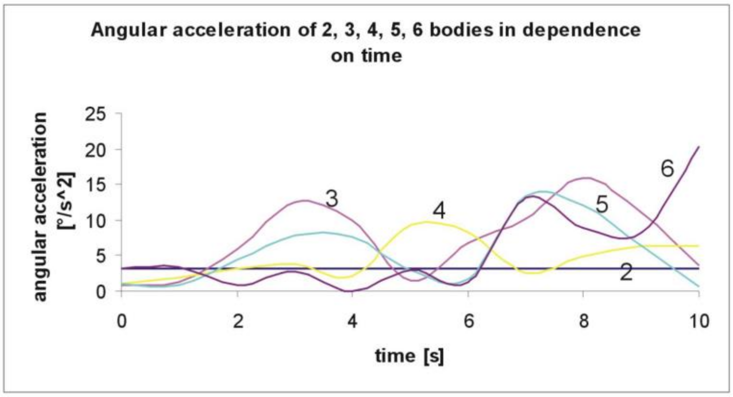
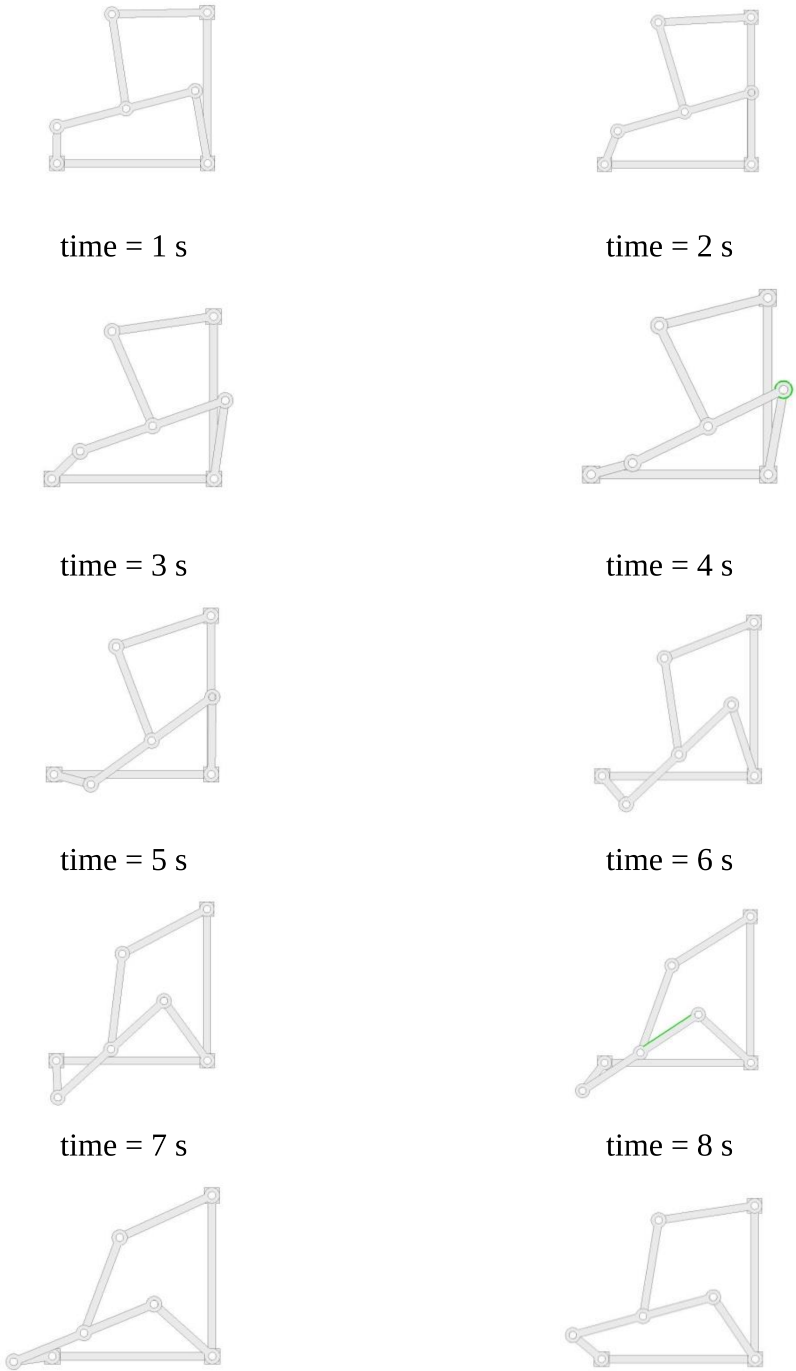


Fig. 6.54 Angular acceleration of 2, 3, 4, 5, 6 bodies in dependence on the time

Relating to planar mechanism, the simulation of operation can be seen in the Fig. 6.55 for time step referring to one second while the whole simulation takes place for ten seconds.



time = 1 s time = 2 s

time = 3 s time = 4 s

time = 5 s time = 6 s

time = 7 s time = 8 s

time = 9 s time = 10 s

Fig. 6.55 Simulation of planar mechanism operation for ten positions

The whole course of the velocity (speed) and acceleration for C, D, E, F points of bodies can be seen in Fig.6.56 and Fig.6.57.

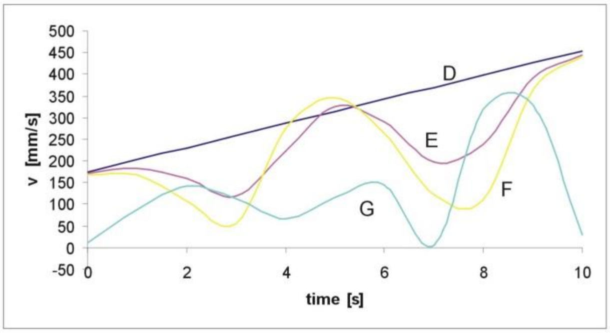


Fig. 6.56 Velocity (speed) in points (D, E, F, G) – in dependence on the time

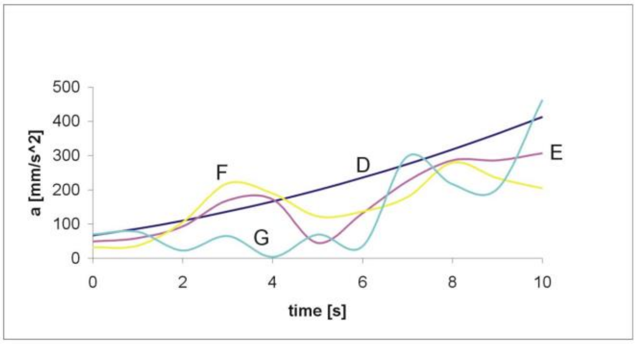


Fig. 6.57 Acceleration in points (D, E, F, G) – in dependence on the time

The main objective of the dynamic analysis is connected with specification of the loading for the individual items (members or bodies) and determination of the courses relating to mutual reactions, referring to individual kinematic connections. The analysis was based on utilisation of the linear model. Relating to the analysis, the other important values were utilised:

- modulus of elasticity (Young's modulus): E = 210 [GPa],
- Poisson's ratio: $\mu = 0.3$,
- density of material: $\rho = 7850$ [kg.m^{-3}].

Fig. 6.58 represents the course of the reaction in D point of the body, designated as 2 and Fig.6.59 represents the course of the reaction in C point of the body, designated as 1.

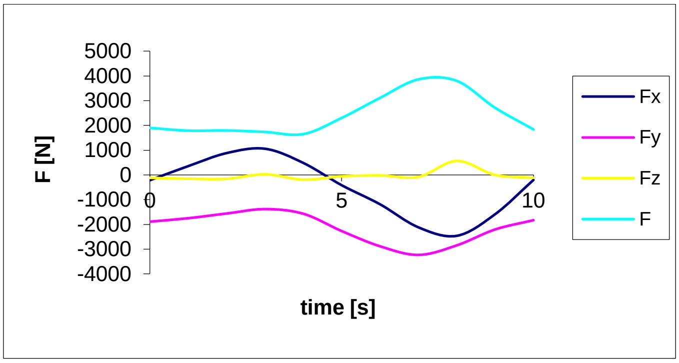


Fig. 6.58 Course of the reaction in D point of the body, designated as 2 – in dependence on the time

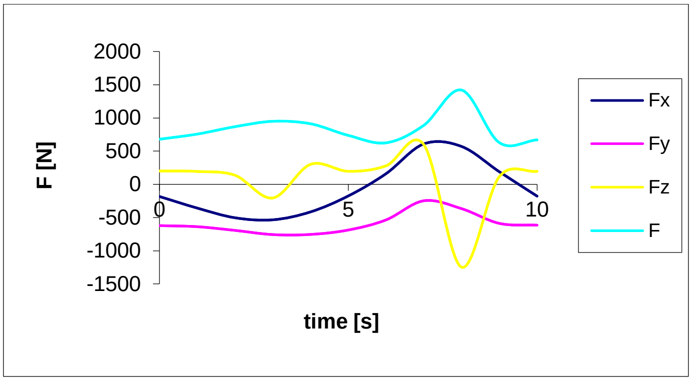


Fig. 6.59 Course of the reaction in C point of the body, designated as 1 – in dependence on the time

The distribution of the stress for bodies, designated as 1, 2, 3, 4 can be seen in Figs. from 6.60 to 6.67.

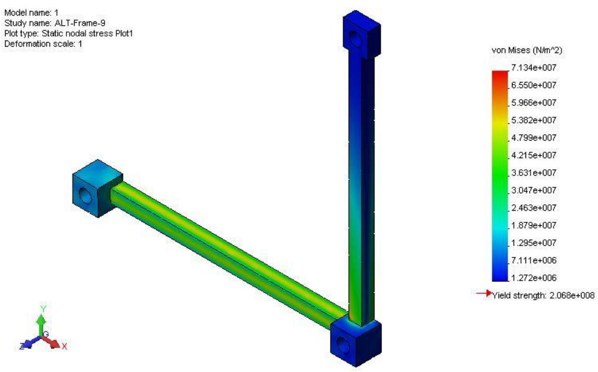


Fig. 6.60 Distribution of the stresses [Pa] for body, designated as 1

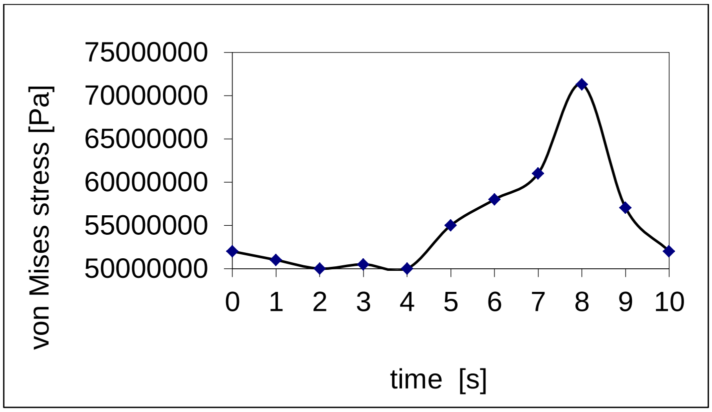


Fig. 6.61 Course of the stresses for body, designated as 1 – in dependence on the time

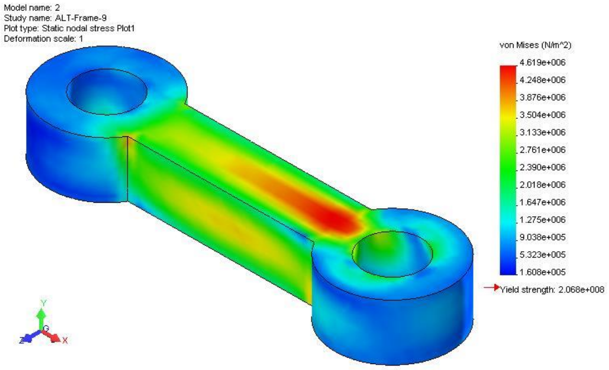


Fig. 6.62 Distribution of the stresses [Pa] for body, designated as 2

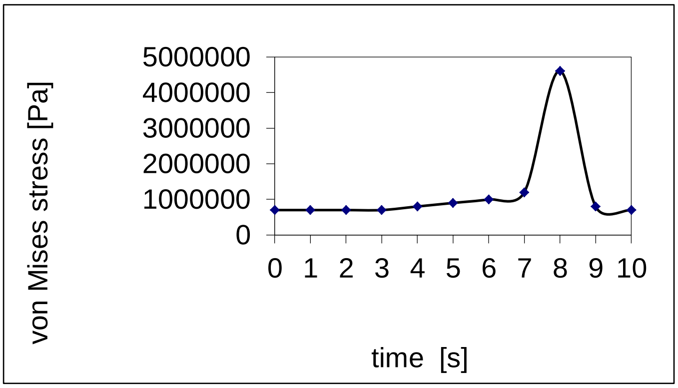


Fig. 6.63 Course of the stresses for body, designated as 2 – in dependence on the time

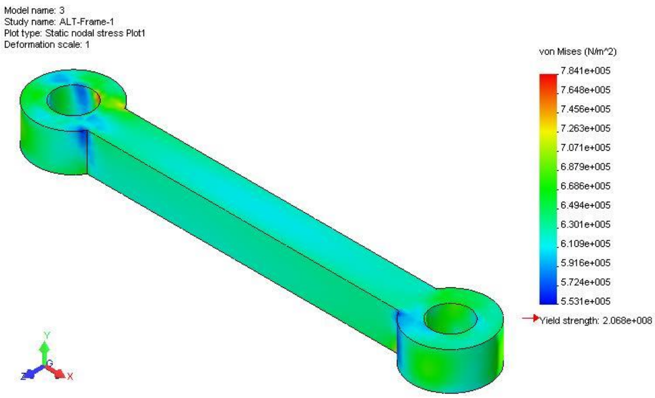


Fig. 6.64 Distribution of the stresses [Pa] for body, designated as 3

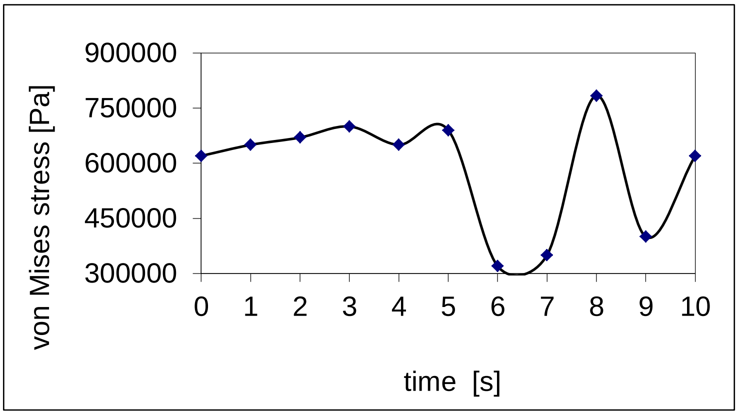


Fig. 6.65 Course of the stresses for body, designated as 3 – in dependence on the time

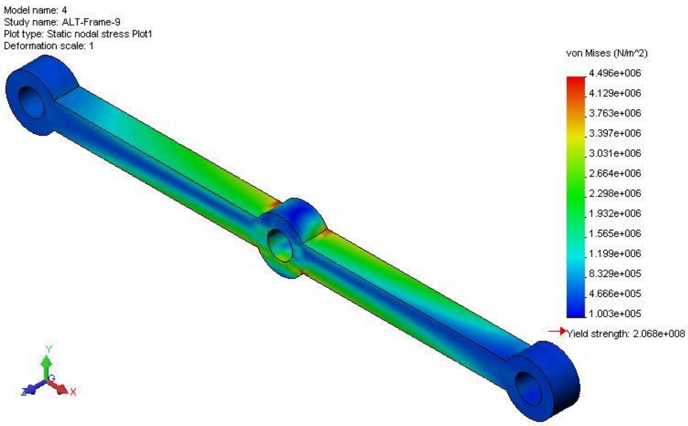


Fig. 6.66 Course of the stresses [Pa] for body, designated as 4

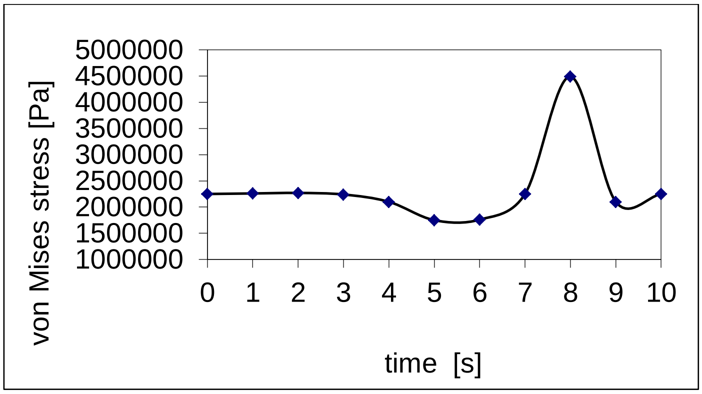


Fig. 6.67 Course of the stresses for body, designated as 4 – in dependence on the time

Example 2

The planar mechanism (Fig. 6.68) represents mechanism consisting of six bodies. Using the kinematic analysis and dynamic analysis and subsequent simulation [31], the main objective is connected with the determination and entering of the position domains, speed (velocity) domains as well as acceleration of the individual bodies in relation to the specified input values of the angular velocity for the driving body designated as 2. The angular velocity for body, designated as 2 is specified: $\omega_{21} = 36\, s^{-1}$.

Input values:

a = 1.7 [m], b = 1.3 [m], c = 0.1 [m], h = 0.1 [m] (thickness of bodies), l_2 = 0.5 [m], l_3 = 0.1 [m], l_4 = 1.4 [m], l_5=1.4 [m], l_6 = 0.9 [m].

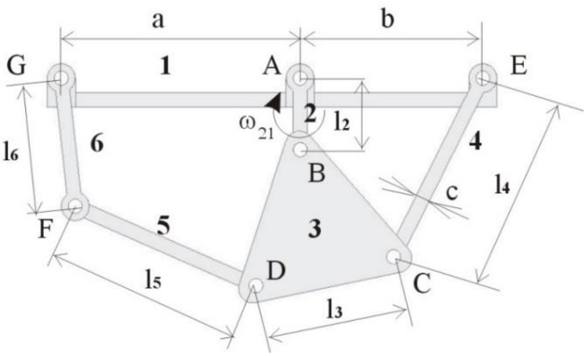


Fig. 6.68 Planar mechanism

The simulation of operation relating to planar mechanism can be seen in the Fig. 6.69 for time step referring to one second while the all simulation takes place for ten seconds.

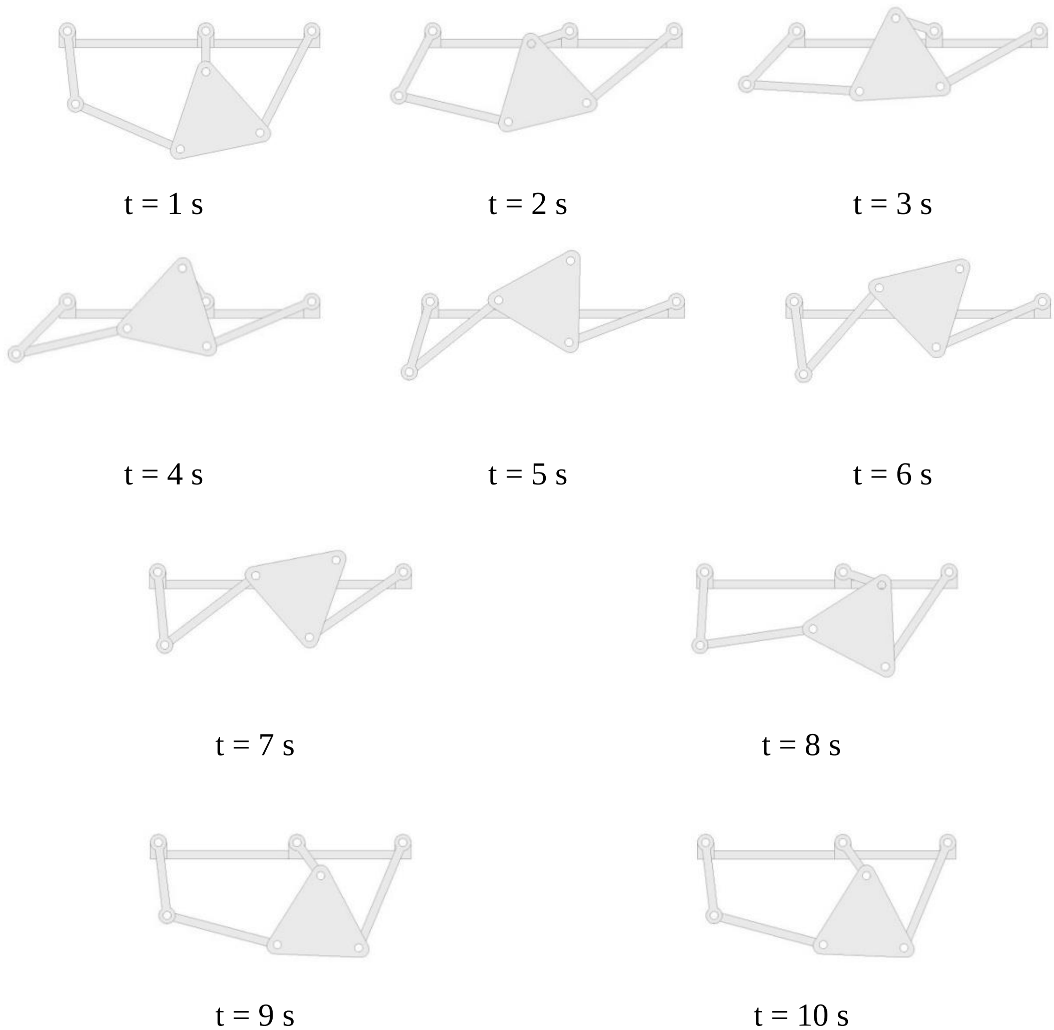


Fig. 6.69 Simulation of planar mechanism operation

The whole course of the velocity and acceleration for B, C, D, F points of bodies can be seen in Fig.6.70 and Fig.6.71.

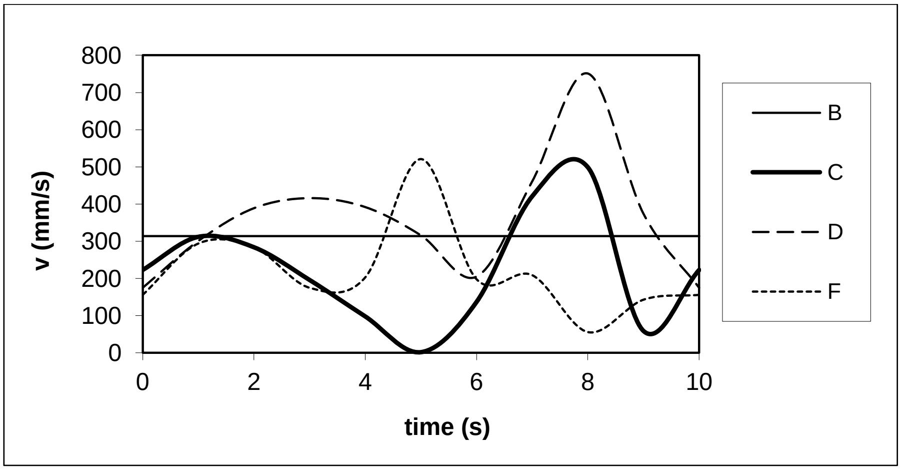


Fig. 6.70 Velocity (speed) of B, C, D, F points in dependence on the time

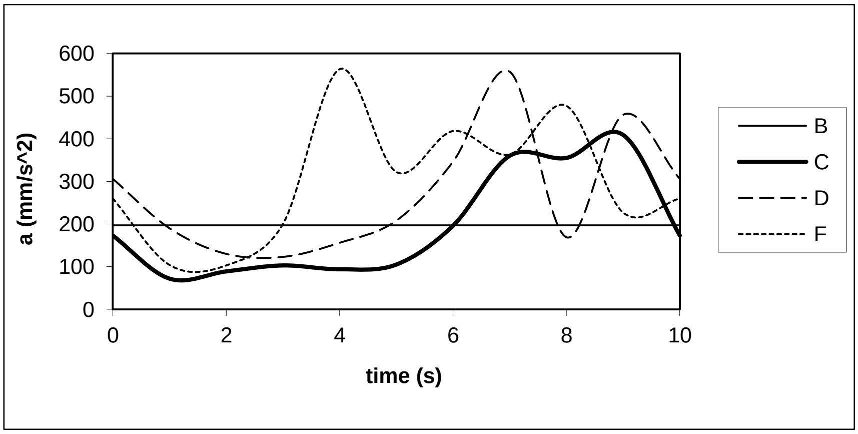


Fig. 6.71 Acceleration of B, C, D, F points in dependence on the time

The computational model of the planar mechanism can be seen in the Fig. 8.72.

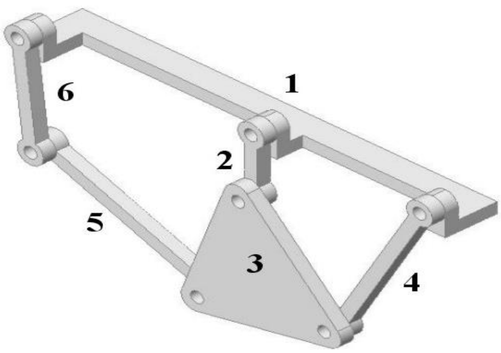


Fig. 6.72 Computational model of the planar mechanism

The main objective of the dynamic analysis is connected with specification of the loading for the individual items and determination of the courses relating to mutual reactions for individual kinematic connections. The analysis was based on utilisation of the linear model. Relating to the analysis, the other important and utilised input values are:

- modulus of elasticity (Young's modulus): $E = 2.1\ e^{11}$ (Pa),
- Poisson's ratio: $\mu = 0.3$,
- density of material: $\rho = 7850$ (kg.m^{-3}).

Fig. 6.73 represents the course of the reaction in E point of the body, designated as 4 and Fig. 6.74 represents the course of the reaction in F point of the body, designated as 6.

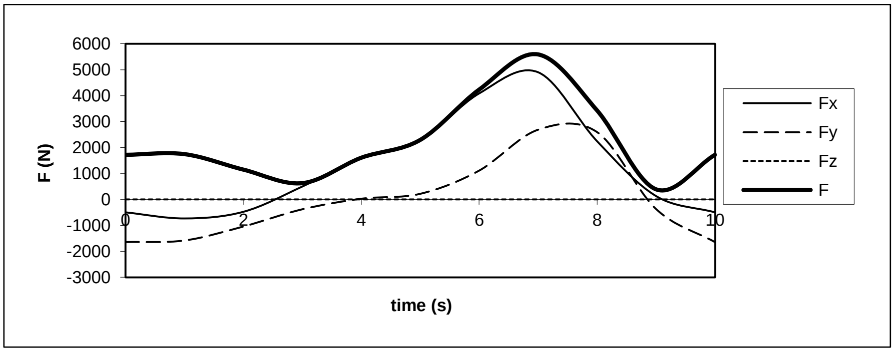


Fig. 6.73 Course of the reaction in E point of the body, designated as 4 in dependence on the time

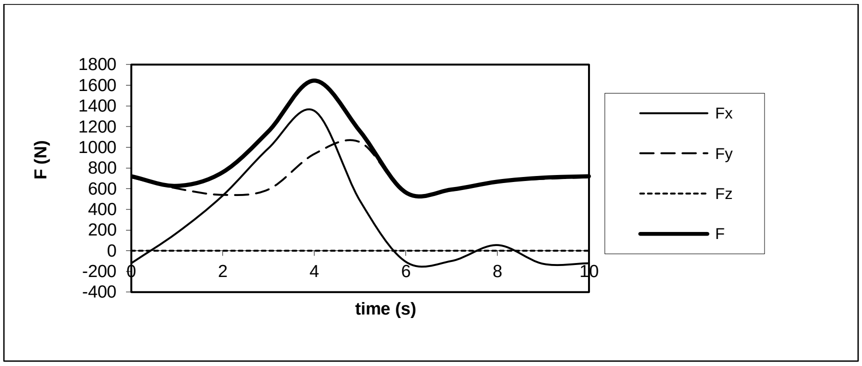


Fig. 6.74 Course of the reaction in F point of the body, designated as 6 in dependence on the time

Moreover, the distribution of the stress for bodies, designated as 1, 3, 6, can be seen in Figs. 6.75-6.77.

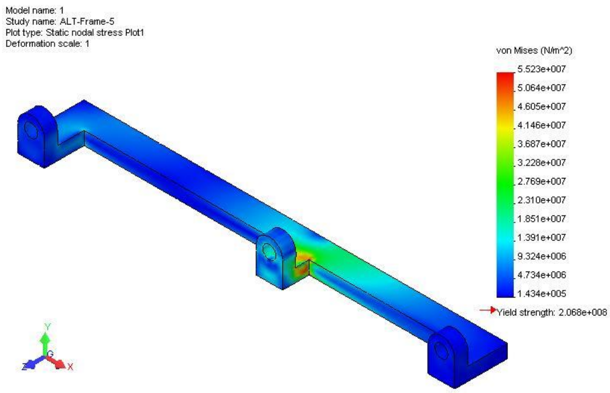


Fig. 6.75 Distribution of the stress for body, designated as 1

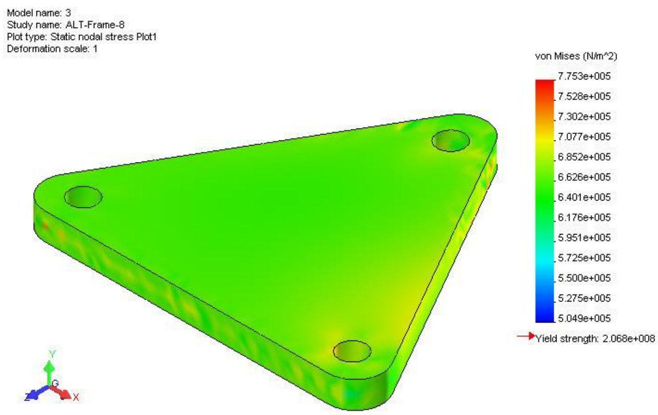


Fig. 6.76 Distribution of the stress for body, designated as 3

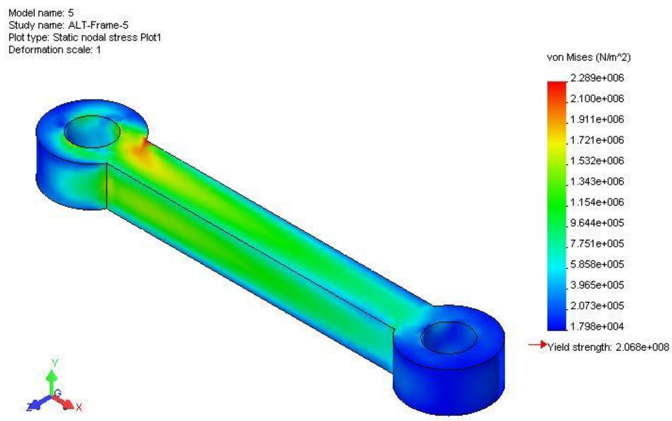


Fig. 6.77 Distribution of the stress for body, designated as 6

Example 3

The planar mechanism representative (Fig. 6.78) consists of six bodies and it was used as computational model. Using the kinematic analysis and dynamic analysis and subsequent simulation [29], the main objective is connected with the determination and entering of the position domains, speed (velocity) domains as well as acceleration of the individual bodies in relation to the specified input values of the angular velocity for the driving body, designated as 2. The angular velocity for body, designated as 2, is specified in this way: ω_{21}=20 [°/s] and α_{21}=3.2 [°/s^2], where ω_{21}=20 [°/s] is not constant and it is changed in dependence on time (Fig. 6.79).

Specified input values:

a = 1.5 [m], b = 1.5 [m], c = 0.1 [m], h = 0.1 [m] (thickness of bodies), l_2 = 0.5 [m], l_3 = 1,4 [m], l_4 = 1.6 [m], l_5=1.3 [m], l_6 = 2,2 [m].

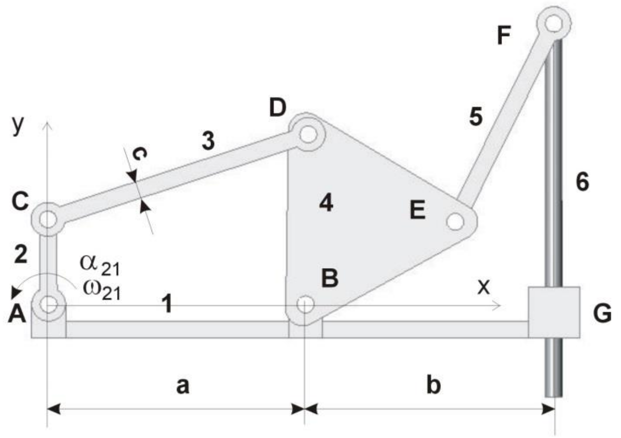


Fig. 6.78 Planar mechanism – computational model

Course of input value for angular velocity and angular acceleration can be seen in Fig. 6.79 and Fig. 6.80.

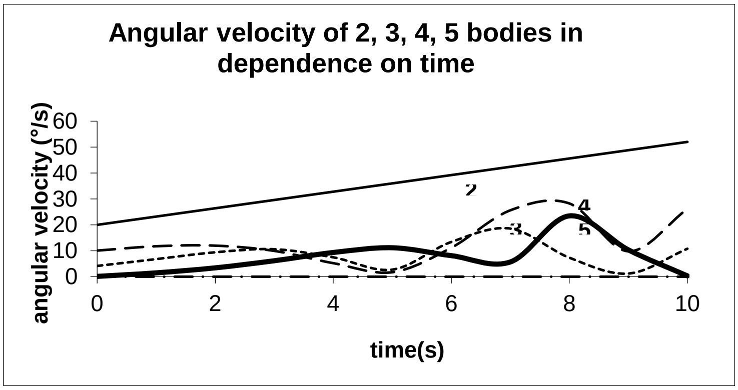


Fig. 6.79 Angular velocity of 2, 3, 4, 5 bodies in dependence on the time

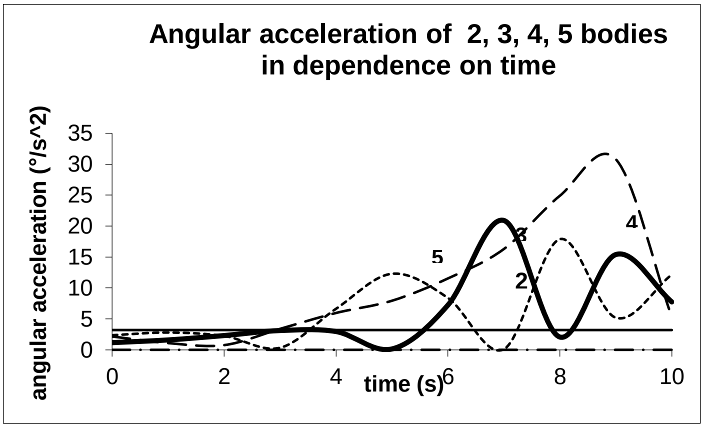


Fig. 6.80 Angular acceleration of 2, 3, 4, 5 bodies in dependence on the time

The simulation of planar mechanism operation for four positions can be seen in the Fig. 6.81.

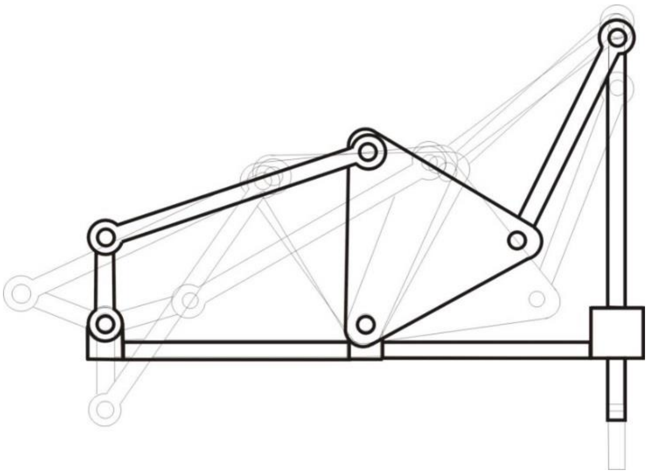


Fig. 6.81 Simulation of planar mechanism operation for four positions

The whole course of the velocity (speed) and acceleration for C, D, F points of bodies can be seen in Fig. 6.82 and Fig. 6.83.

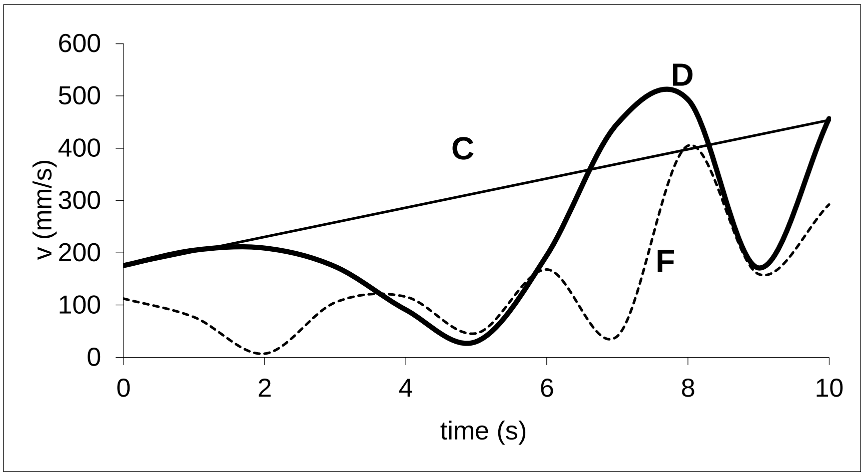


Fig. 6.82 Velocity in points (C, D, F) – in dependence on the time

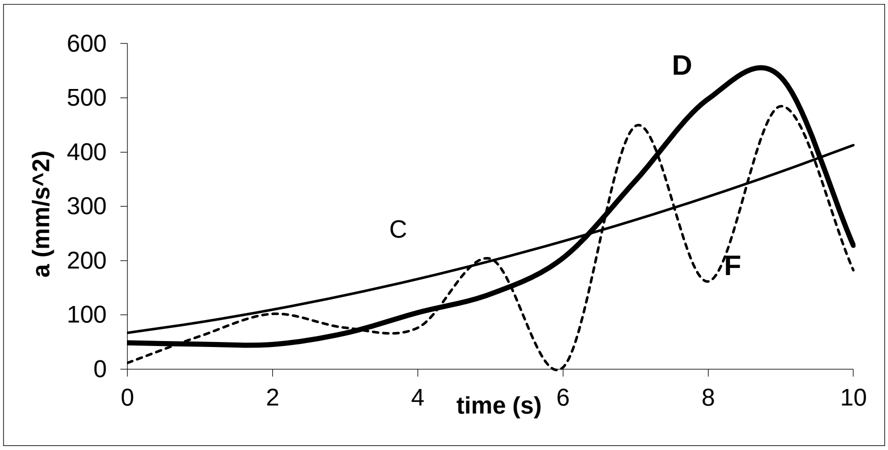


Fig. 6.83 Acceleration in points (C, D, F) – in dependence on the time

The main objective of the dynamic analysis is connected with specification of the loading for the individual items and determination of the courses relating to mutual reactions for individual kinematic connections. The analysis was based on utilisation of the linear model. Relating to the analysis, the other important and utilised values were:

- modulus of elasticity (Young's modulus): $E = 2.1\ e^{11}$ (Pa),
- Poisson's ratio: $\mu = 0.3$,
- density of material: $\rho = 7850$ ($kg.m^{-3}$).

Fig. 6.84 represents the course of the reaction in E point of the body, designated as 4 and Fig. 6.85 represents the course of the reaction in D point of the body, designated as 3.

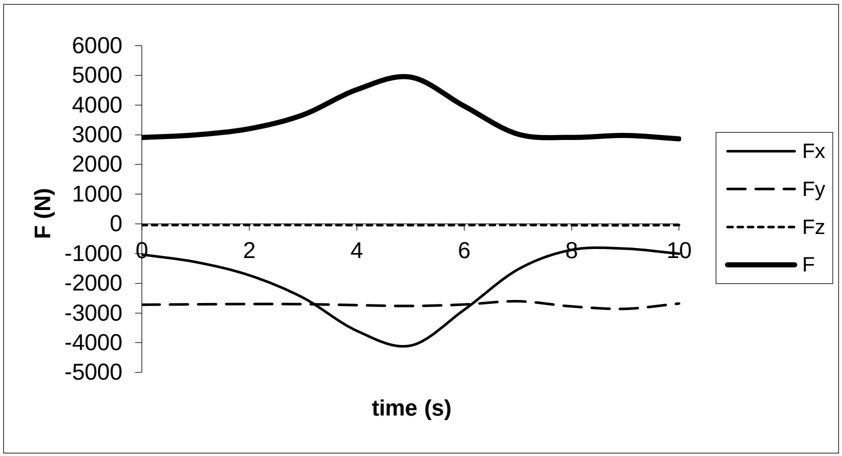


Fig. 6.84 Course of the reaction in E point of the body, designated as 4 – in dependence on the time

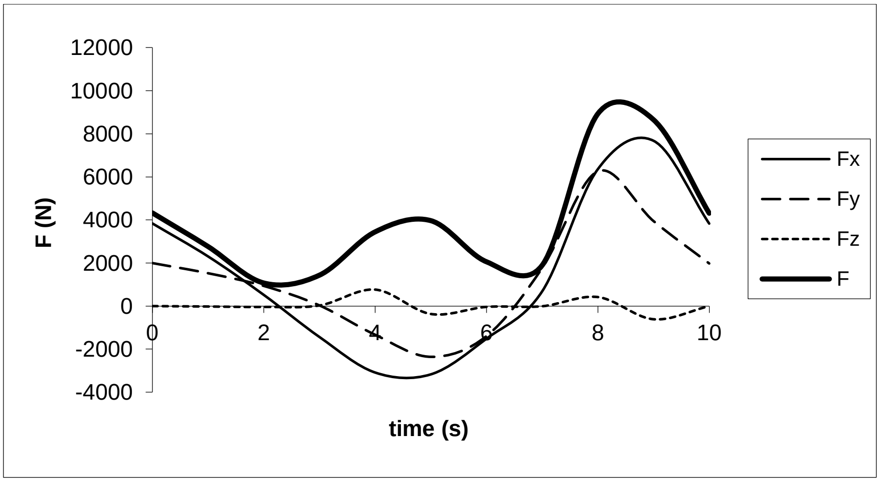


Fig. 6.85 Course of the reaction in D point of the body, designated as 3 – in dependence on the time

The distribution of the stress for connected bodies, designated as 1, 4, 5, 3, can be seen in Figs. 6.86-6.93.

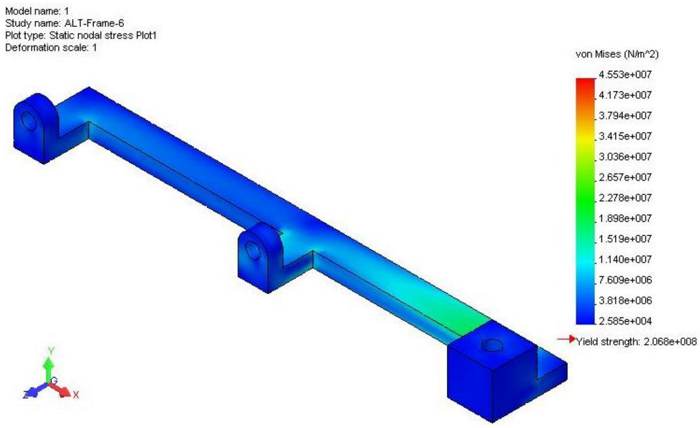


Fig. 6.86 Distribution of the stress for body, designated as 1 in [Pa]

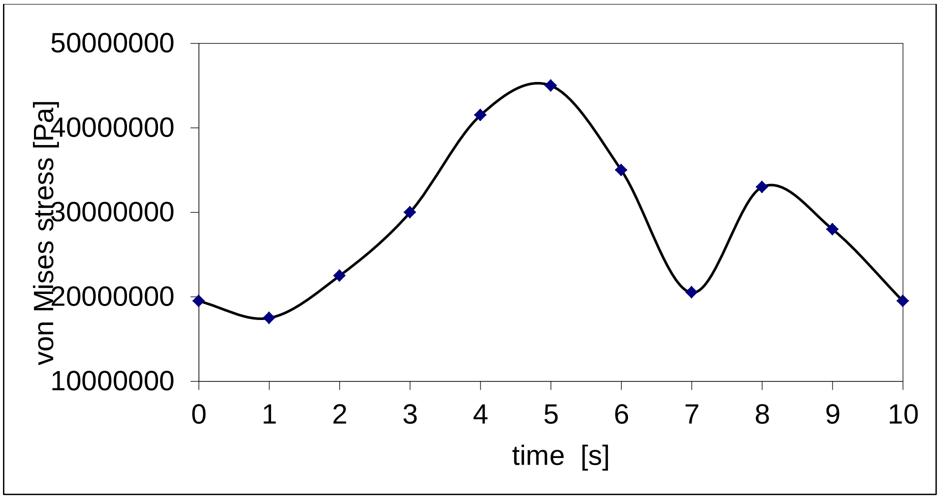


Fig. 6.87 Course of the stress for body designated as 1 – in dependence on the time

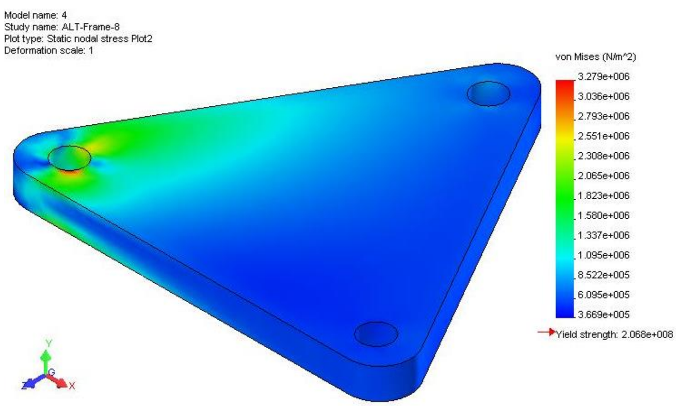


Fig. 6.88 Distribution of the stress for body, designated as 4 in [Pa]

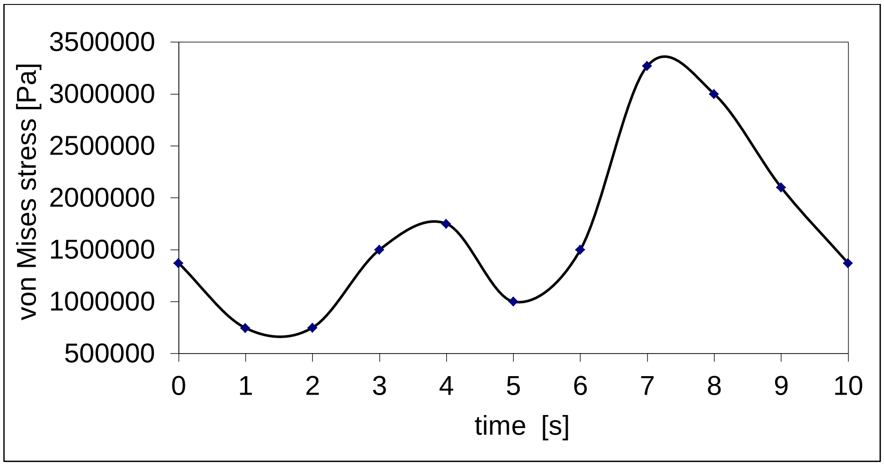


Fig. 6.89 Course of the stress for body, designated as 4 – in dependence on the time

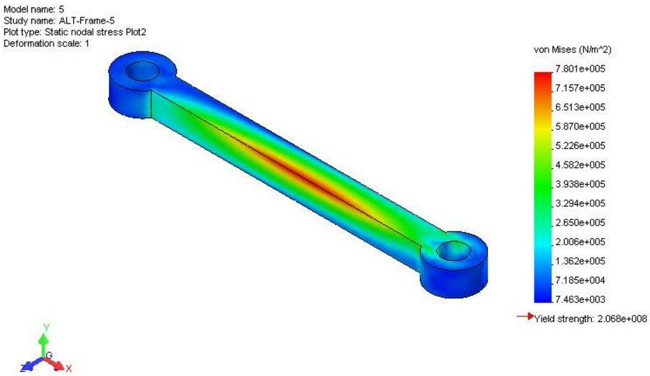


Fig. 6.90 Distribution of the stress for body designated as 5 in [Pa]

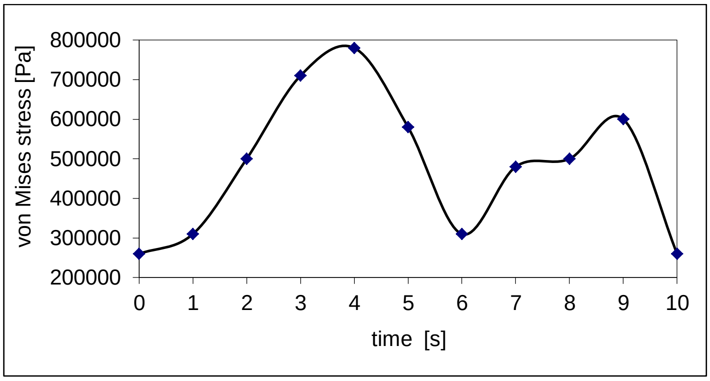


Fig. 6.91 Course of the stress for body designated as 5 – in dependence on the time

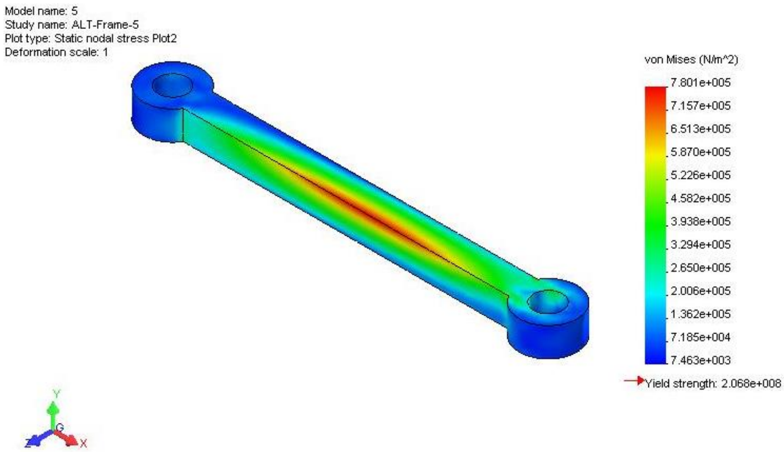


Fig. 6.92 Distribution of the stress for body, designated as 3 in [Pa]

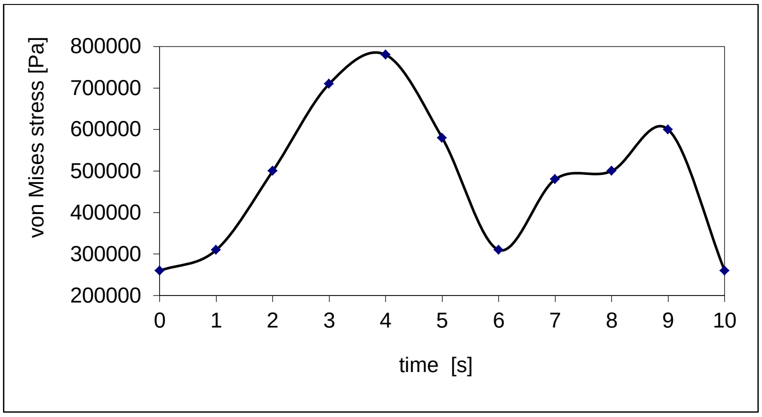


Fig. 6.93 Course of the for body, designated as 3 – in dependence on the time

Example 4

The planar mechanism representative (Fig. 6.94) consists of five bodies and it was used as computational model. Using the kinematic analysis [1-2] and subsequent simulation [7–9], [15], the main objective is connected with the determination and entering of the position domains, speed (velocity) domains as well as acceleration of the individual bodies. In relation to the five-item mechanism (mechanism with the five members), the field of positions, speeds (velocities) and accelerations is going to be solved by the vector method. The numerical solution is carried out for the specified input values (6.1) if the specified angle of rotational displacement for the driving item (member), designated as 2, is: $\varphi_{21} = 210^\circ$ and the angular velocity for the body, designated as 2 is ω_{21}=10 rad.s$_{-1}$ = konst. The kinematic scheme of which can be seen in Fig. 6.95. Formation of the equations of the position (6.3), (6.4), (6.5) the differentiation of which, for $\phi_i\,(1,2,3,4,5,6)$ variables (6.2), gives the specific equations for the speeds (velocities) (6.6) and then the given equations are rewritten into a matrix form (6.7). If the equations (6.6) are differentiated once more, we can get equations (6.8) for the accelerations and the given equations can be rewritten into a matrix form (6.9).

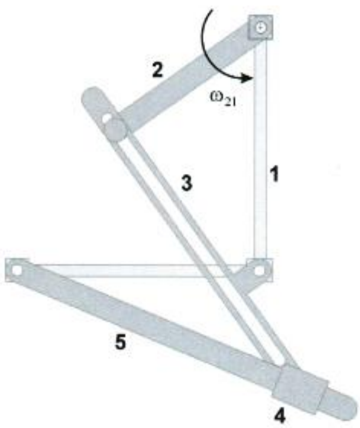


Fig. 6.94 Computational model

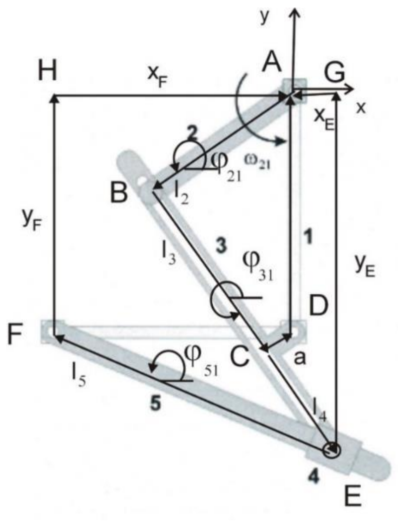


Fig. 6.95 Kinematic scheme of the given mechanism

Input parameters:

$AB = l_2 = 0.04\ m,\ BC = l_3 = 0.05\ m,\ EF = l_5 = 0.08\ m,\ BE = 0.08\ m,\ CE = l_4 = 0.03\ m,$
$y_F = 0.05\ m,\ x_E = 0.05\ m,\ a = 0.005\ m$
$\varphi_{21} = 210^\circ,\ \varphi_{31} = 225^\circ,\ \varphi_{51} = 140^\circ,$
$\omega_{21} = 10\ rad.s^{-1} = konst$ (6.1)

The variables are:

$$[\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6] = [\varphi_{31}, l_3, x_E, y_E, \varphi_{51}, l_5] \tag{6.2}$$

The equations of the position for the ABCDA loop are:

$$\begin{aligned} f_1 &= l_2.\cos\varphi_{21} + l_3.\cos\varphi_{31} + a_x = 0 \\ f_2 &= l_2.\sin\varphi_{21} + l_3.\sin\varphi_{31} + a_y + y_F = 0 \end{aligned} \tag{6.3}$$

The equations of the position for the ABEGA loop are:

$$\begin{aligned} f_3 &= l_2.\cos\varphi_{21} + (l_3 + l_4).\cos\varphi_{31} - x_E = 0 \\ f_4 &= l_2.\sin\varphi_{21} + (l_3 + l_4).\sin\varphi_{31} + y_E = 0 \end{aligned} \tag{6.4}$$

The equations of the position for the ABEFHA loop are:

$$\begin{aligned} f_5 &= l_2.\cos\varphi_{21} + (l_3 + l_4).\cos\varphi_{31} + l_5.\cos\varphi_{51} + x_F = 0 \\ f_6 &= l_2.\sin\varphi_{21} + (l_3 + l_4).\sin\varphi_{31} + l_5.\sin\varphi_{51} + y_F = 0 \end{aligned} \tag{6.5}$$

$$
\begin{aligned}
&\dot{f}_1 = -l_2.\dot{\varphi}_{21}\sin\varphi_{21} + \dot{l}_3.\cos\varphi_{31} - l_3.\dot{\varphi}_{31}\sin\varphi_{31} = 0\\
&\dot{f}_2 = l_2.\dot{\varphi}_{21}\cos\varphi_{21} + \dot{l}_3.\sin\varphi_{31} + l_3.\dot{\varphi}_{31}\cos\varphi_{31} = 0\\
&\dot{f}_3 = -l_2.\dot{\varphi}_{21}\sin\varphi_{21} + \dot{l}_3\cos\varphi_{31} - (l_3 + l_4).\dot{\varphi}_{31}\sin\varphi_{31} - \dot{x}_E = 0\\
&\dot{f}_4 = l_2.\dot{\varphi}_{21}\cos\varphi_{21} + \dot{l}_3\sin\varphi_{31} + (l_3 + l_4).\dot{\varphi}_{31}\cos\varphi_{31} + \dot{y}_E = 0\\
&\dot{f}_5 = -l_2.\dot{\varphi}_{21}\sin\varphi_{21} + \dot{l}_3\cos\varphi_{31} - (l_3 + l_4).\dot{\varphi}_{31}\sin\varphi_{31} + \dot{l}_5\cos\varphi_{51} - l_5\dot{\varphi}_{51}\sin\varphi_{51} = 0\\
&\dot{f}_6 = l_2.\dot{\varphi}_{21}\cos\varphi_{21} + \dot{l}_3\sin\varphi_{31} + (l_3 + l_4).\dot{\varphi}_{31}\cos\varphi_{31} + \dot{l}_5\sin\varphi_{51} + l_5\dot{\varphi}_{51}\cos\varphi_{51} = 0
\end{aligned}
\tag{6.6}
$$

$$
\begin{bmatrix}
-l_3\sin\varphi_{31} & \cos\varphi_{31} & 0 & 0 & 0 & 0\\
l_3\cos\varphi_{31} & \sin\varphi_{31} & 0 & 0 & 0 & 0\\
-(l_3+l_4)\sin\varphi_{31} & \cos\varphi_{31} & -1 & 0 & 0 & 0\\
(l_3+l_4)\cos\varphi_{31} & \sin\varphi_{31} & 0 & 1 & 0 & 0\\
-(l_3+l_4)\sin\varphi_{31} & \cos\varphi_{31} & 0 & 0 & -l_5\sin\varphi_{51} & \cos\varphi_{51}\\
(l_3+l_4)\cos\varphi_{31} & \sin\varphi_{31} & 0 & 0 & l_5\cos\varphi_{51} & \sin\varphi_{51}
\end{bmatrix}
.
\begin{bmatrix}
\dot{\varphi}_{31}\\ \dot{l}_3\\ \dot{x}_E\\ \dot{y}_E\\ \dot{\varphi}_{51}\\ \dot{l}_5
\end{bmatrix}
=
\begin{bmatrix}
l_2\sin\varphi_{21}\\ -l_2\cos\varphi_{21}\\ l_2\sin\varphi_{21}\\ -l_2\cos\varphi_{21}\\ l_2\sin\varphi_{21}\\ -l_2\cos\varphi_{21}
\end{bmatrix}
.\dot{\varphi}_{21}
\tag{6.7}
$$

$$
\begin{aligned}
&\ddot{f}_1 = -l_2.\dot{\varphi}_{21}^2\cos\varphi_{21} + \ddot{l}_3.\cos\varphi_{31} - \dot{l}_3.\dot{\varphi}_{31}\sin\varphi_{31} - \dot{l}_3.\dot{\varphi}_{31}\sin\varphi_{31} - l_3.\ddot{\varphi}_{31}\sin\varphi_{31} -\\
&- l_3.\dot{\varphi}_{31}^2\cos\varphi_{31} = 0\\
&\ddot{f}_2 = -l_2.\dot{\varphi}_{21}^2\sin\varphi_{21} + \ddot{l}_3.\sin\varphi_{31} + \dot{l}_3.\dot{\varphi}_{31}\cos\varphi_{31} + \dot{l}_3.\dot{\varphi}_{31}\cos\varphi_{31} + l_3.\ddot{\varphi}_{31}\cos\varphi_{31} -\\
&- l_3.\dot{\varphi}_{31}^2\sin\varphi_{31} = 0\\
&\ddot{f}_3 = -l_2.\dot{\varphi}_{21}^2\cos\varphi_{21} + \ddot{l}_3.\cos\varphi_{31} - \dot{l}_3.\dot{\varphi}_{31}\sin\varphi_{31} - (l_3 + l_4).\dot{\varphi}_{31}^2\cos\varphi_{31} -\\
&- (l_3 + l_4).\ddot{\varphi}_{31}\sin\varphi_{31} - l_3.\dot{\varphi}_{31}\sin\varphi_{31} - \ddot{x}_E = 0\\
&\ddot{f}_4 = -l_2.\dot{\varphi}_{21}^2\sin\varphi_{21} + \ddot{l}_3.\sin\varphi_{31} + \dot{l}_3.\dot{\varphi}_{31}\cos\varphi_{31} - (l_3 + l_4).\dot{\varphi}_{31}^2\sin\varphi_{31} -\\
&- (l_3 + l_4).\ddot{\varphi}_{31}\cos\varphi_{31} - l_3.\dot{\varphi}_{31}\cos\varphi_{31} - \ddot{y}_E = 0\\
&\ddot{f}_5 = -l_2.\dot{\varphi}_{21}^2\cos\varphi_{21} + \ddot{l}_3.\cos\varphi_{31} - \dot{l}_3.\dot{\varphi}_{31}\sin\varphi_{31} - (l_3 + l_4).\dot{\varphi}_{31}^2\cos\varphi_{31} -\\
&- (l_3 + l_4).\ddot{\varphi}_{31}\sin\varphi_{31} - l_3.\dot{\varphi}_{31}\sin\varphi_{31} + \ddot{l}_5.\cos\varphi_{51} - \dot{l}_5.\dot{\varphi}_{51}\sin\varphi_{51} - \dot{l}_5.\dot{\varphi}_{51}\sin\varphi_{51} -\\
&- l_5.\ddot{\varphi}_{51}\sin\varphi_{51} - l_5.\dot{\varphi}_{51}^2\cos\varphi_{51} = 0\\
&\ddot{f}_6 = -l_2.\dot{\varphi}_{21}^2\sin\varphi_{21} + \ddot{l}_3.\sin\varphi_{31} + \dot{l}_3.\dot{\varphi}_{31}\cos\varphi_{31} - (l_3 + l_4).\dot{\varphi}_{31}^2\sin\varphi_{31} -\\
&- (l_3 + l_4).\ddot{\varphi}_{31}\cos\varphi_{31} - l_3.\dot{\varphi}_{31}\cos\varphi_{31} + \ddot{l}_5.\sin\varphi_{51} + \dot{l}_5.\dot{\varphi}_{51}\cos\varphi_{51} + \dot{l}_5.\dot{\varphi}_{51}\cos\varphi_{51} +\\
&+ l_5.\ddot{\varphi}_{51}\cos\varphi_{51} - l_5.\dot{\varphi}_{51}^2\sin\varphi_{51} = 0
\end{aligned}
\tag{6.8}
$$

$$\begin{bmatrix} -l_3\sin\varphi_{31} & \cos\varphi_{31} & 0 & 0 & 0 & 0 \\ l_3\cos\varphi_{31} & \sin\varphi_{31} & 0 & 0 & 0 & 0 \\ -(l_3+l_4)\sin\varphi_{31} & \cos\varphi_{31} & -1 & 0 & 0 & 0 \\ (l_3+l_4)\cos\varphi_{31} & \sin\varphi_{31} & 0 & 1 & 0 & 0 \\ -(l_3+l_4)\sin\varphi_{31} & \cos\varphi_{31} & 0 & 0 & -l_5\sin\varphi_{51} & \cos\varphi_{51} \\ (l_3+l_4)\cos\varphi_{31} & \sin\varphi_{31} & 0 & 0 & l_5\cos\varphi_{51} & \sin\varphi_{51} \end{bmatrix} \cdot \begin{bmatrix} \ddot{\varphi}_{31} \\ \ddot{l}_3 \\ \ddot{x}_E \\ \ddot{y}_E \\ \ddot{\varphi}_{51} \\ \ddot{l}_5 \end{bmatrix} = \begin{bmatrix} l_2\sin\varphi_{21} \\ -l_2\cos\varphi_{21} \\ l_2\sin\varphi_{21} \\ -l_2\cos\varphi_{21} \\ l_2\sin\varphi_{21} \\ -l_2\cos\varphi_{21} \end{bmatrix} . \dot{\varphi}_{21}^2 + \tag{6.9}$$

$$+ \begin{bmatrix} l_3\cos\varphi_{31} \\ -l_3\sin\varphi_{31} \\ (l_3+l_4)\cos\varphi_{31} \\ (l_3+l_4)\sin\varphi_{31} \\ (l_3+l_4)\cos\varphi_{31} \\ (l_3+l_4)\sin\varphi_{31} \end{bmatrix} . \dot{\varphi}_{31}^2 + \begin{bmatrix} 2\dot{l}_3\sin\varphi_{31} \\ -2\dot{l}_3\cos\varphi_{31} \\ \dot{l}_3\sin\varphi_{31} \\ -\dot{l}_3\cos\varphi_{31} \\ \dot{l}_3\sin\varphi_{31} \\ -\dot{l}_3\cos\varphi_{31} \end{bmatrix} . \dot{\varphi}_{31} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2\dot{l}_5\sin\varphi_{51} \\ -2\dot{l}_5\cos\varphi_{51} \end{bmatrix} . \dot{\varphi}_{51} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ l_5\cos\varphi_{51} \\ l_5\sin\varphi_{51} \end{bmatrix} . \dot{\varphi}_{51}^2$$

The results from the kinematic analysis

The given solution was carried out by using the SolidWorks and the results of the kinematic analysis are shown in Figs. 6.96, 6.97 and the animation for the first four positions is shown in Fig. 6.98.

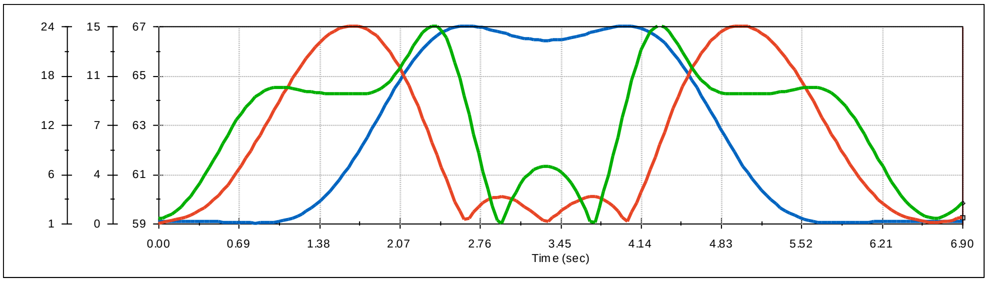


Fig. 6.96 Course of the position (blue), velocity (red) and acceleration (green) for the body 3 of the mechanism

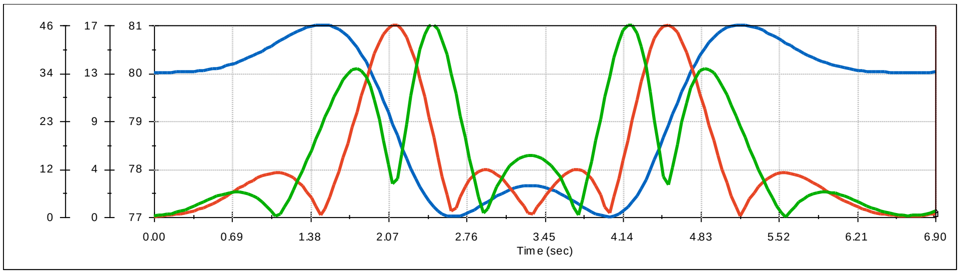


Fig. 6.97 Course of the position (blue), velocity (red) and acceleration (green) for the body 5 of the mechanism

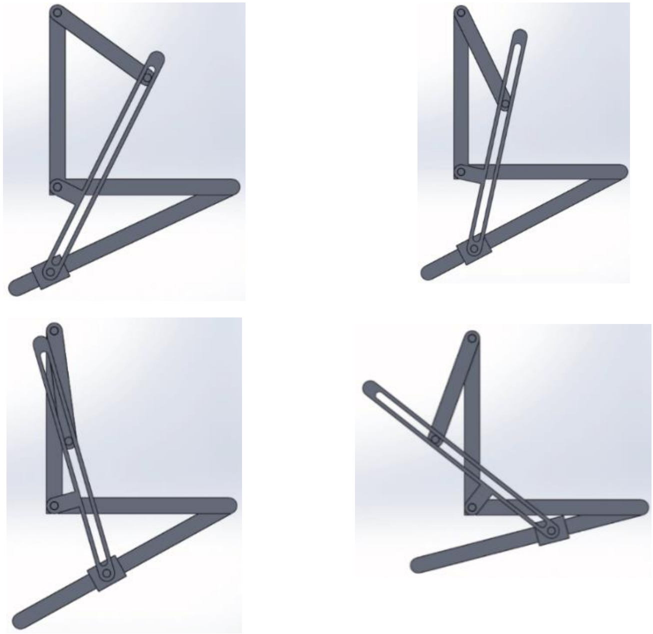


Fig. 6.98 Animation of the four positions of the mechanism

The analysis of the planar mechanism is based on selection of the linear tetrahedral element with four nodes (see chapter 2.2).

The main objective of the dynamic analysis is connected with specification of the loading for the individual items and determination of the courses relating to mutual reactions, referring to individual kinematic connections [8], [10-14], [16]. The analysis was based on utilisation of the nonlinear model. Relating to the analysis, the other important values were utilised:

- modulus of elasticity (Young's modulus): E = 210 GPa,
- Poisson's ratio: $\mu = 0.3$,
- density of material: ρ = 7850 kg m^{-3}.

Fig. 6.99 shows the degraded degrees of freedom of the given mechanism.

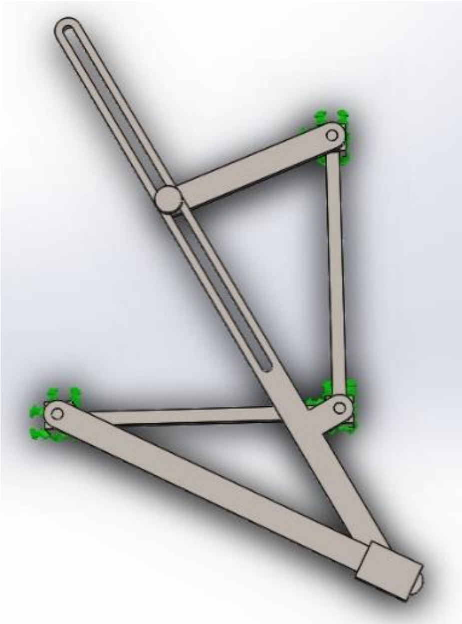


Fig. 6.99 Degraded degrees of freedom of the mechanism

The network of finite elements of the planar mechanism can be seen in Fig. 6.100.

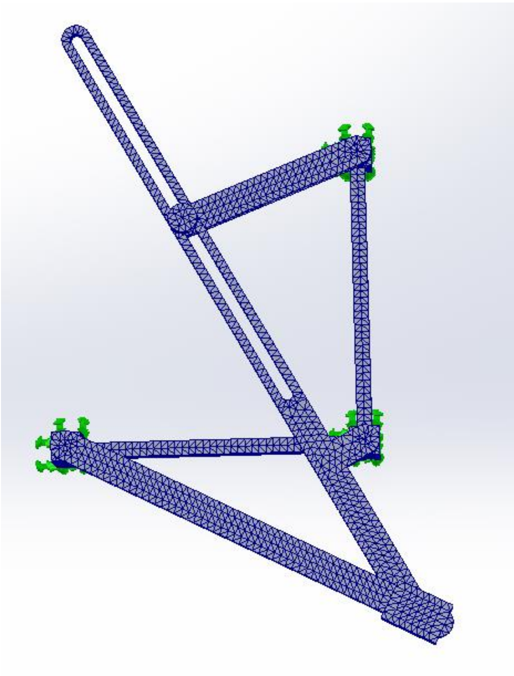


Fig. 6.100 Finite element network

The distribution of the stress for mechanism [17-18] can be seen in Fig. 6.101 and distribution of the displacement for mechanism is shown in Fig. 6.102.

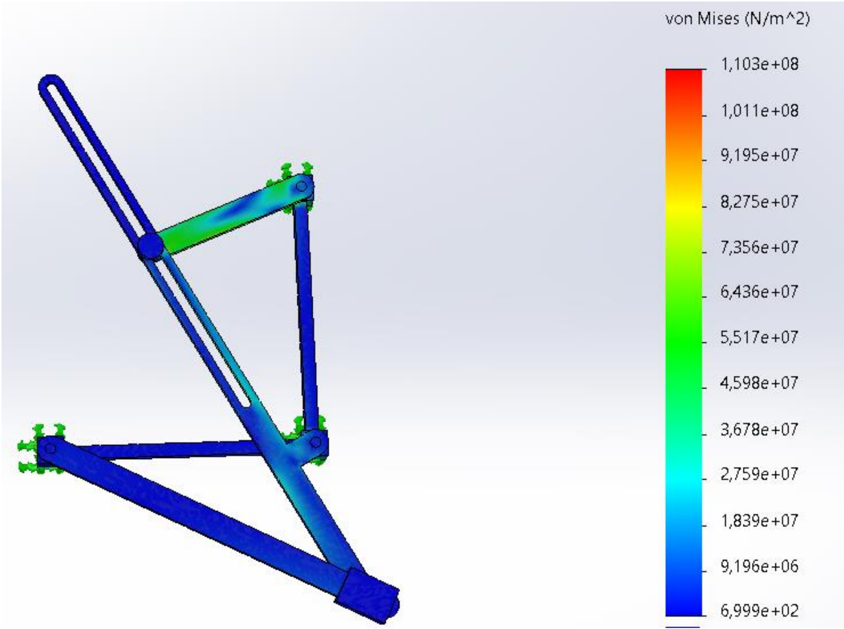


Fig. 6.101 Distribution of the stresses (Pa) for the mechanism

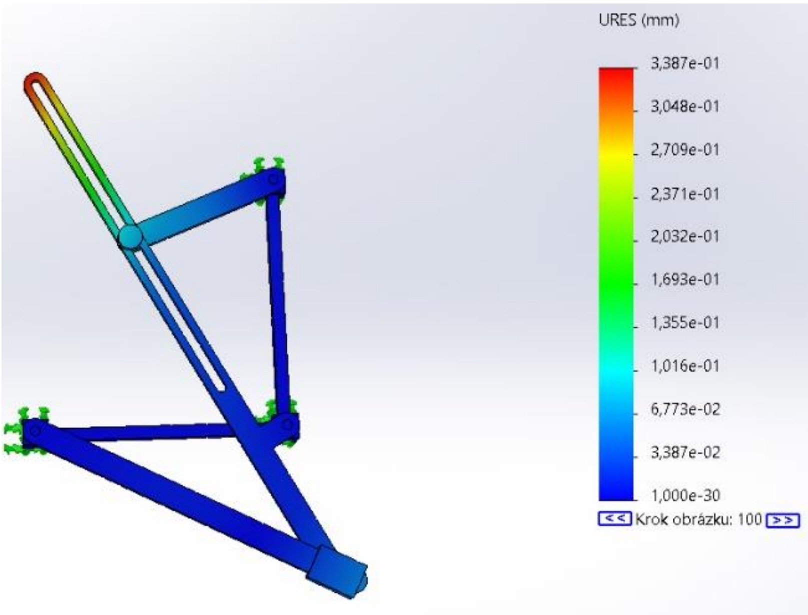


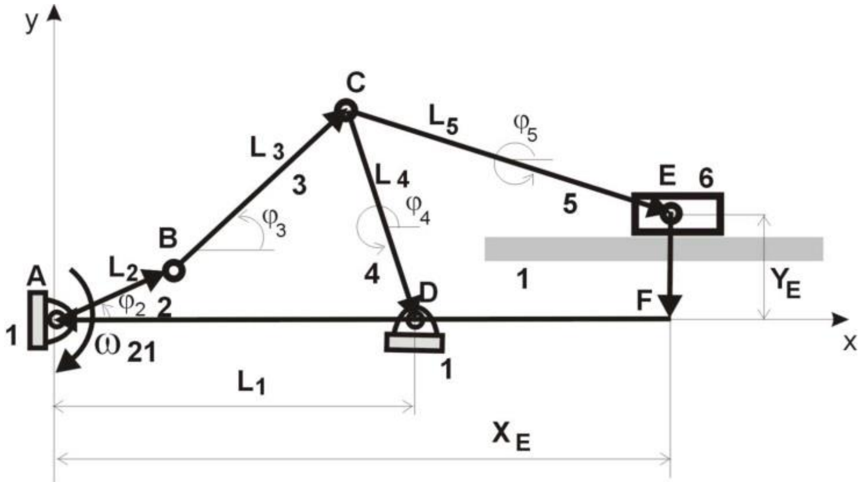
Fig. 6.102 Distribution of the displacement (mm) for the mechanism

7 Procedures for Kinematic and Dynamic Analysis of Planar Mechanisms by Means of SolidWorks Software

The following chapter includes the introduction and description of the procedures which are closely connected with kinematic and dynamic analysis of a six-item body system (Fig.7.1), using the SolidWorks software [36].

Input parameters:

$AB = L_2 = 0.045 \quad m,\ BC = L_3 = 0.095 \quad m,\ CD = L_4 = 0.09 \quad m,$
$CE = L_5 = 0.06 \quad m,\ DA = L_1 = 0.1 \quad m,\ EF = Y_E = 0.048 \quad m,$
$q = \varphi_2 = 20\ °, \varphi_3 = 60\ °, \varphi_4 = 270\ °,\ \varphi_5 = 343\ °, \omega_{21} = 36\ \deg .s^{-1} = konst \cdot$



Obr. 7.1 Kinematic scheme of mechanism

The variables are:

$$[\phi_1, \phi_2, \phi_3, \phi_4] = [\varphi_3, \varphi_4, \varphi_5, x_E] \tag{7.1}$$

The equations of the position for the ABCDA loop are:

$$\begin{aligned} f_1 &: L_2 \cos\varphi_2 + L_3 \cos\varphi_3 + L_4 \cos\varphi_4 - L_1 = 0 \\ f_2 &: L_2 \sin\varphi_2 + L_3 \sin\varphi_3 + L_4 \sin\varphi_4 = 0 \end{aligned} \tag{7.2}$$

The equations of the position for the ABCEFA loop are:

$$
\begin{aligned}
f_3 &: L_2 \cos\varphi_2 + L_3 \cos\varphi_3 + L_5 \cos\varphi_5 - x_E = 0 \\
f_4 &: L_2 \sin\varphi_2 + L_3 \sin\varphi_3 + L_5 \sin\varphi_5 - y_E = 0
\end{aligned}
\tag{7.3}
$$

If the equations of the positions (7.2) and (7.3) are differentiated on the basis of $\phi_i (i = 1,2,3,4)$ variables, the system of velocity equations (7.4) is obtained and after further differentiation, the system of acceleration equations (7.5) can be obtained.

$$
\begin{aligned}
\dot{f}_1 &: -L_2\dot{\varphi}_2 \sin\varphi_2 - L_3\dot{\varphi}_3 \sin\varphi_3 - L_4\dot{\varphi}_4 \sin\varphi_4 = 0 \\
\dot{f}_2 &: L_2\dot{\varphi}_2 \cos\varphi_2 + L_3\dot{\varphi}_3 \cos\varphi_3 + L_4\dot{\varphi}_4 \cos\varphi_4 = 0 \\
\dot{f}_3 &: -L_2\dot{\varphi}_2 \sin\varphi_2 - L_3\dot{\varphi}_3 \sin\varphi_3 - L_5\dot{\varphi}_5 \sin\varphi_5 - \dot{x}_E = 0 \\
\dot{f}_4 &: L_2\dot{\varphi}_2 \cos\varphi_2 + L_3\dot{\varphi}_3 \cos\varphi_3 + L_5\dot{\varphi}_5 \cos\varphi_5 = 0
\end{aligned}
\tag{7.4}
$$

$$
\begin{aligned}
\ddot{f}_1 &: -L_2\dot{\varphi}_2^2 \cos\varphi_2 - L_3\ddot{\varphi}_3 \sin\varphi_3 - L_3\dot{\varphi}_3^2 \cos\varphi_3 - L_4\ddot{\varphi}_4 \sin\varphi_4 - L_4\dot{\varphi}_4^2 \cos\varphi_4 = 0 \\
\ddot{f}_2 &: -L_2\dot{\varphi}_2^2 \sin\varphi_2 + L_3\ddot{\varphi}_3 \cos\varphi_3 - L_3\dot{\varphi}_3^2 \sin\varphi_3 + L_4\ddot{\varphi}_4 \cos\varphi_4 - L_4\dot{\varphi}_4^2 \sin\varphi_4 = 0 \\
\ddot{f}_3 &: -L_2\dot{\varphi}_2^2 \cos\varphi_2 - L_3\ddot{\varphi}_3 \sin\varphi_3 - L_3\dot{\varphi}_3^2 \cos\varphi_3 - L_5\ddot{\varphi}_5 \sin\varphi_5 - L_5\dot{\varphi}_5^2 \cos\varphi_5 - \ddot{x}_E = 0 \\
\ddot{f}_4 &: -L_2\dot{\varphi}_2^2 \sin\varphi_2 + L_3\ddot{\varphi}_3 \cos\varphi_3 - L_3\dot{\varphi}_3^2 \sin\varphi_3 + L_5\ddot{\varphi}_5 \cos\varphi_5 - L_5\dot{\varphi}_5^2 \sin\varphi_5 = 0
\end{aligned}
\tag{7.5}
$$

We rewrite the system of equations (7.4) into the matrix form (7.6) and we rewrite the system of equations (7.5) into the matrix form (7.7).

$$
\begin{vmatrix}
-L_3 \sin\varphi_3 & -L_4 \sin\varphi_4 & 0 & 0 \\
L_3 \cos\varphi_3 & L_4 \cos\varphi_4 & 0 & 0 \\
-L_3 \sin\varphi_3 & 0 & -L_5 \sin\varphi_5 & -1 \\
L_3 \cos\varphi_3 & 0 & L_5 \cos\varphi_5 & 0
\end{vmatrix}
\cdot
\begin{vmatrix}
\dot{\varphi}_3 \\ \dot{\varphi}_4 \\ \dot{\varphi}_5 \\ \dot{x}_E
\end{vmatrix}
=
\begin{vmatrix}
L_2\dot{\varphi}_2 \sin\varphi_2 \\
-L_2\dot{\varphi}_2 \cos\varphi_2 \\
L_2\dot{\varphi}_2 \sin\varphi_2 \\
-L_2\dot{\varphi}_2 \cos\varphi_2
\end{vmatrix}
\tag{7.6}
$$

$$
\begin{vmatrix}
-L_3 \sin\varphi_3 & -L_4 \sin\varphi_4 & 0 & 0 \\
L_3 \cos\varphi_3 & L_4 \cos\varphi_4 & 0 & 0 \\
-L_3 \sin\varphi_3 & 0 & -L_5 \sin\varphi_5 & -1 \\
L_3 \cos\varphi_3 & 0 & L_5 \cos\varphi_5 & 0
\end{vmatrix}
\cdot
\begin{vmatrix}
\ddot{\varphi}_3 \\ \ddot{\varphi}_4 \\ \ddot{\varphi}_5 \\ \ddot{x}_E
\end{vmatrix}
=
\begin{vmatrix}
L_2\dot{\varphi}_2^2 \cos\varphi_2 + L_3\dot{\varphi}_3^2 \cos\varphi_3 + L_4\dot{\varphi}_4^2 \cos\varphi_4 \\
L_2\dot{\varphi}_2^2 \sin\varphi_2 + L_3\dot{\varphi}_3^2 \sin\varphi_3 + L_4\dot{\varphi}_4^2 \sin\varphi_4 \\
L_2\dot{\varphi}_2^2 \cos\varphi_2 + L_3\dot{\varphi}_3^2 \cos\varphi_3 + L_5\dot{\varphi}_5^2 \cos\varphi_5 \\
L_2\dot{\varphi}_2^2 \sin\varphi_2 + L_3\dot{\varphi}_3^2 \sin\varphi_3 + L_5\dot{\varphi}_5^2 \sin\varphi_5
\end{vmatrix}
\tag{7.7}
$$

7.1 Creation of a computational model in the SolidWorks program

Utilising the drawing technical documentation of individual bodies from Fig.7.3 to Fig. 7.15, the individual procedures or steps of kinematic and dynamic analysis can result in the creation of the computational model (Fig.7.2). Subsequently, the whole set of items or members can be used to create the assembly of bodies with individual kinematic connections (Fig.7.15). All of the important input values, including boundary or critical conditions, material constants, loading and meshing of individual bodies for a given group of elements (Fig. 7.16) as well as the start of the kinematic and dynamic analysis (Fig. 7.17) are predetermined. The results can be obtained in graphic or numerical form.

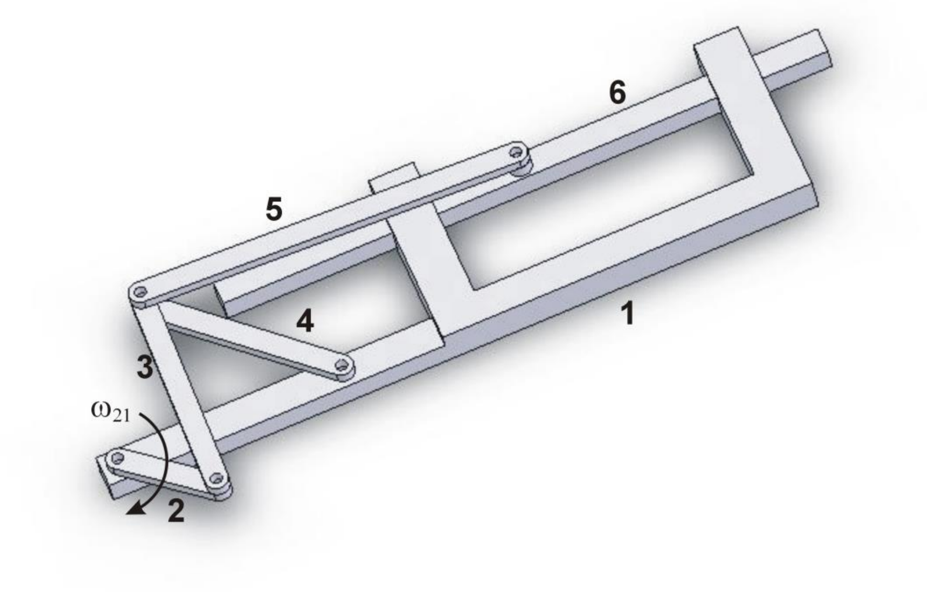


Fig. 7.2 The computational model

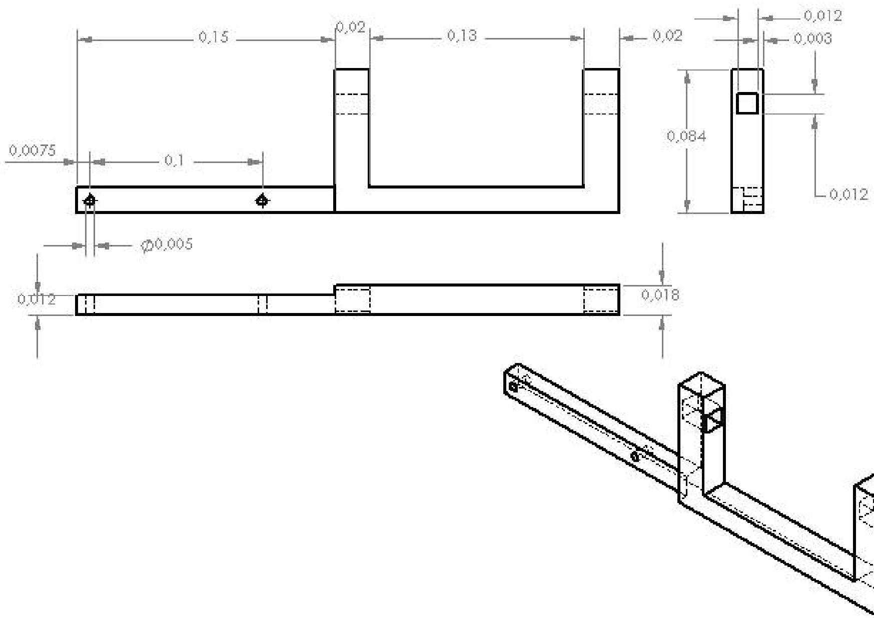


Fig. 7.3 Frame, designated as 1, and its dimensional parameters in [m]

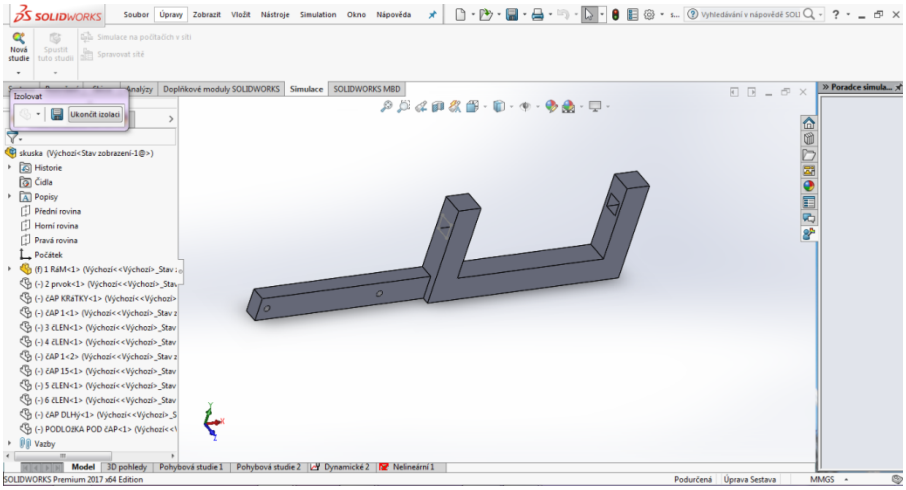


Fig. 7.4 Model of frame in SolidWorks software

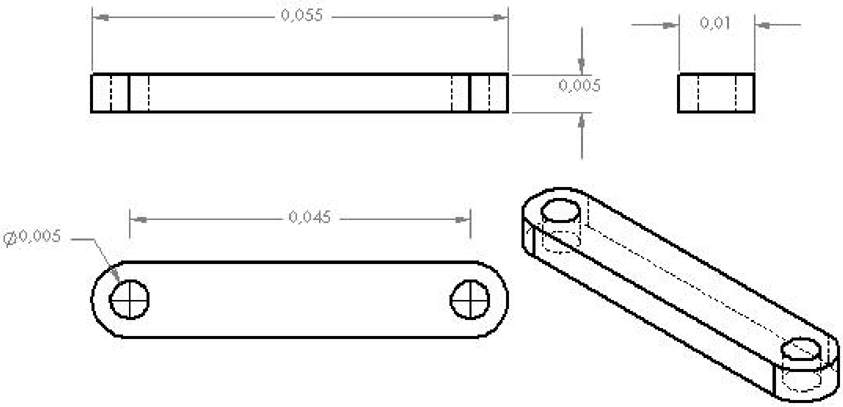


Fig. 7.5 Item or member, designated as 2, and its dimensional parameters in [m]

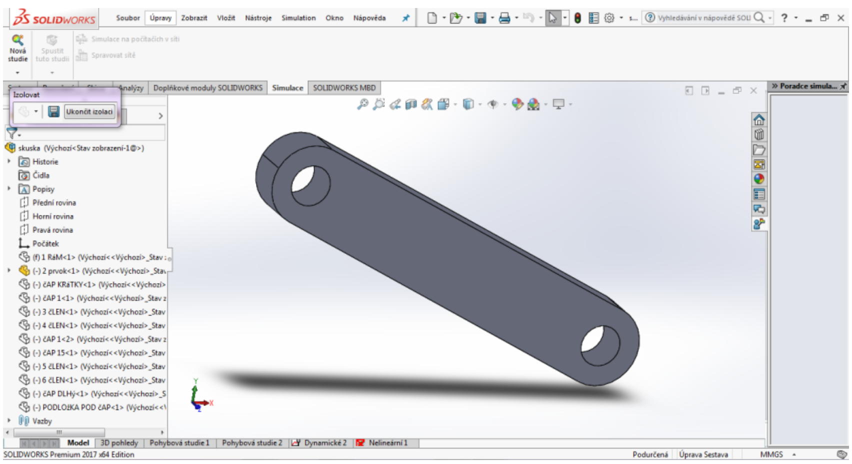


Fig. 7.6 Model of body, designated as 2, in SolidWorks software

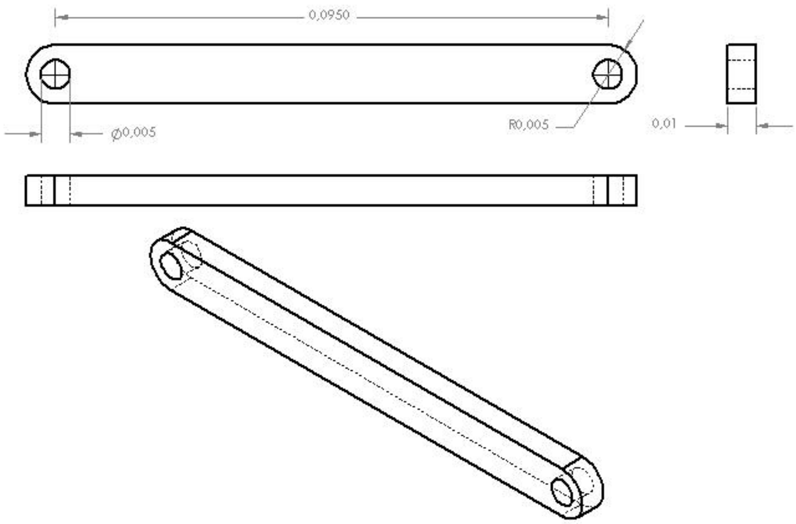


Fig. 7.7 Item (member), designated as 3, and its dimensional parameters in [m]

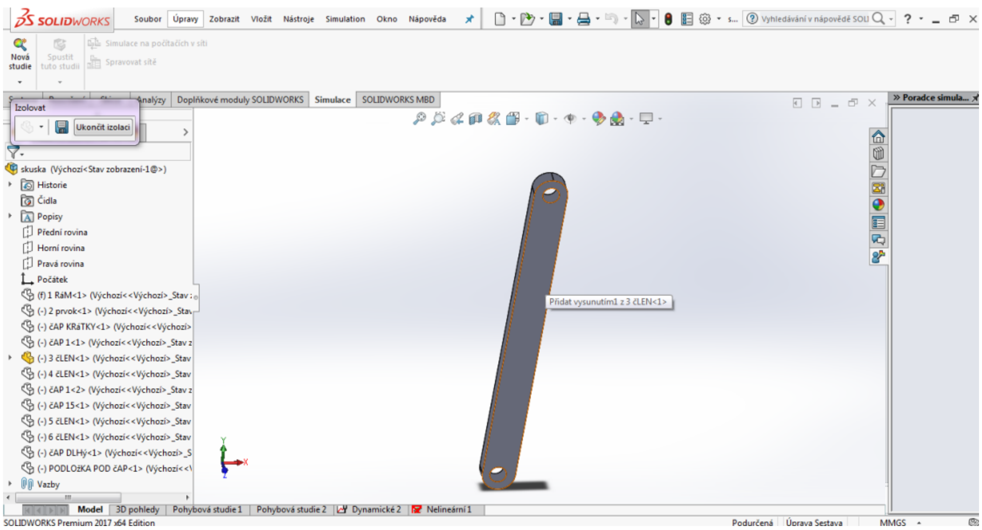


Fig. 7.8 Model of body, designated as 3 in SolidWorks software

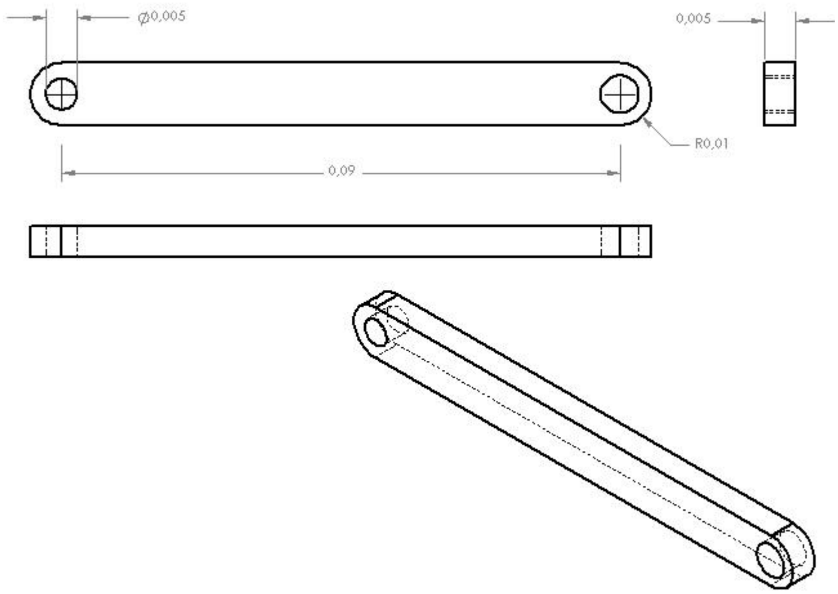


Fig. 7.9 Item or member, designated as 4, and its dimensional parameters in [m]

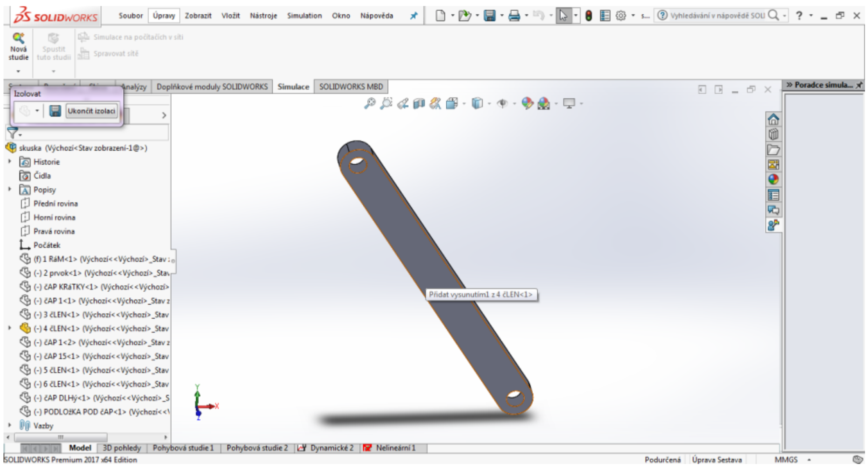


Fig. 7.10 Model of the body, designated as 4 in SolidWorks software

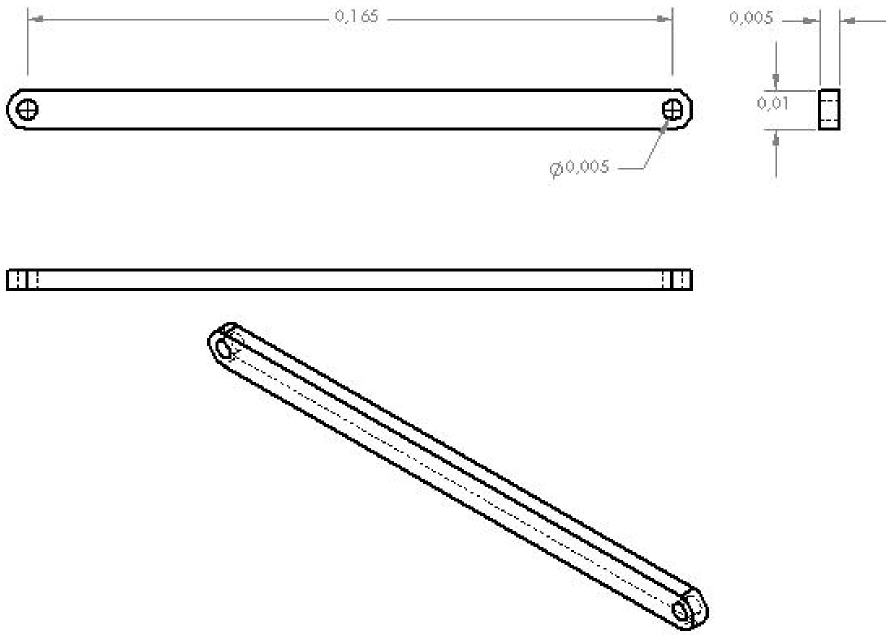


Fig. 7.11 Item or member, designated as 5, and its dimensional parameters in [m]

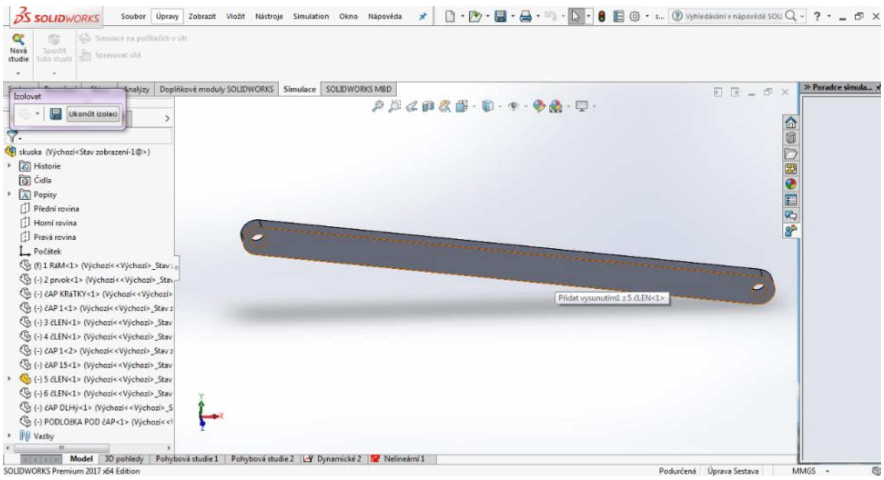


Fig. 7.12 Model of the body, designated as 5 in SolidWorks software

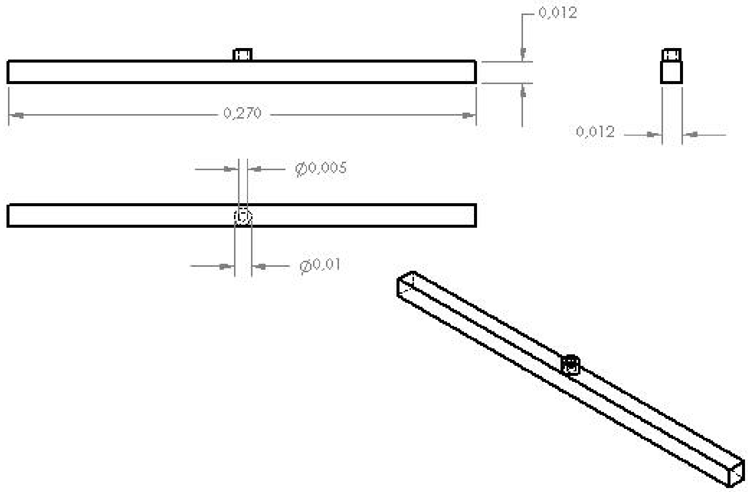


Fig. 7.13 Item or member, designated as 6, and its dimensional parameters in [m]

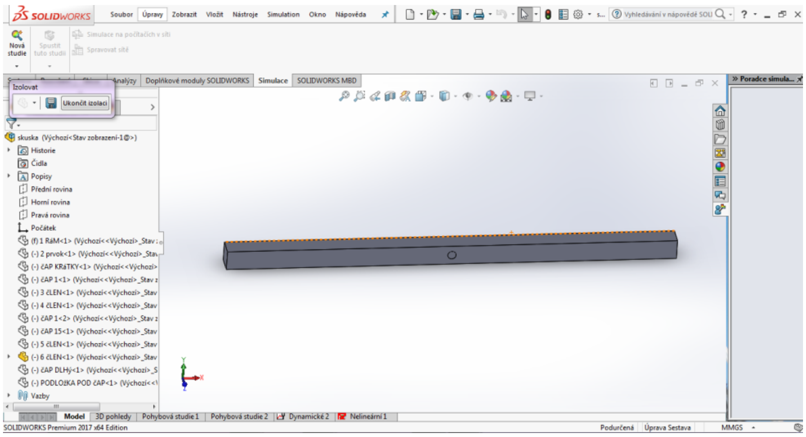


Fig. 7.14 Model of the body, designated as 6 in SolidWorks software

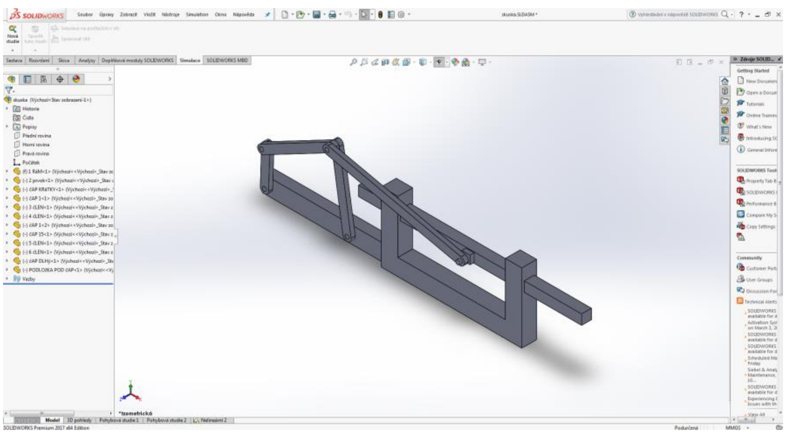


Fig. 7.15 Final assembly of the model with kinematic connections

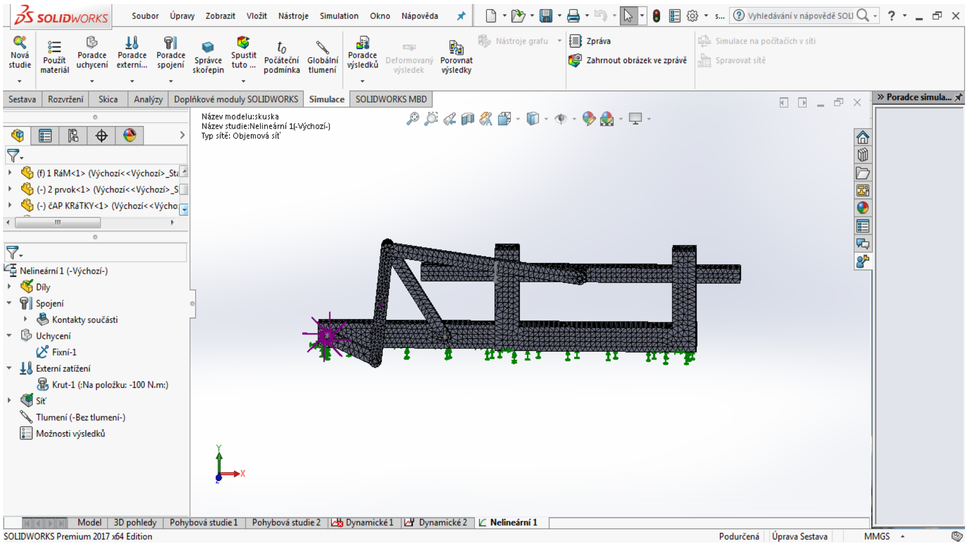


Fig. 7.16 Determination of the boundary (critical) conditions, material constants, loadings and meshing of individual bodies for the given group of elements

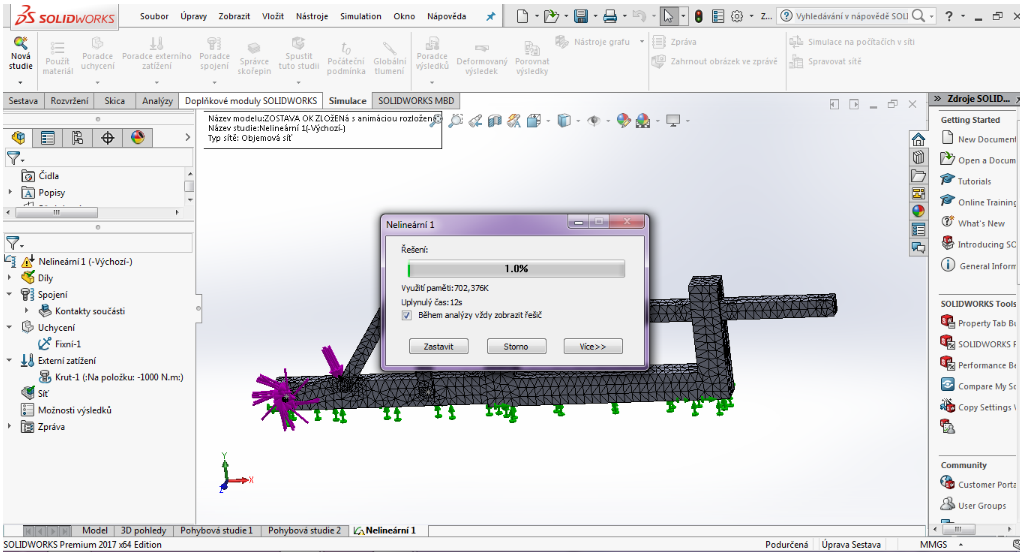


Fig. 7.17 Start of kinematic and dynamic analysis

The results from the kinematic analysis:

Animation of the eight positions of the mechanism can be seen in Fig. 7.18. The course of angular velocities and angular acceleration of the individual bodies of the mechanism is shown in (Fig. 7.19) and (Fig. 7.20). The course of speed and acceleration of individual bodies of the mechanism is shown in (Fig. 7.21) and (Fig. 7.22). On (Fig. 7.23) to (Fig. 7.27) are the results of the dynamic analysis.

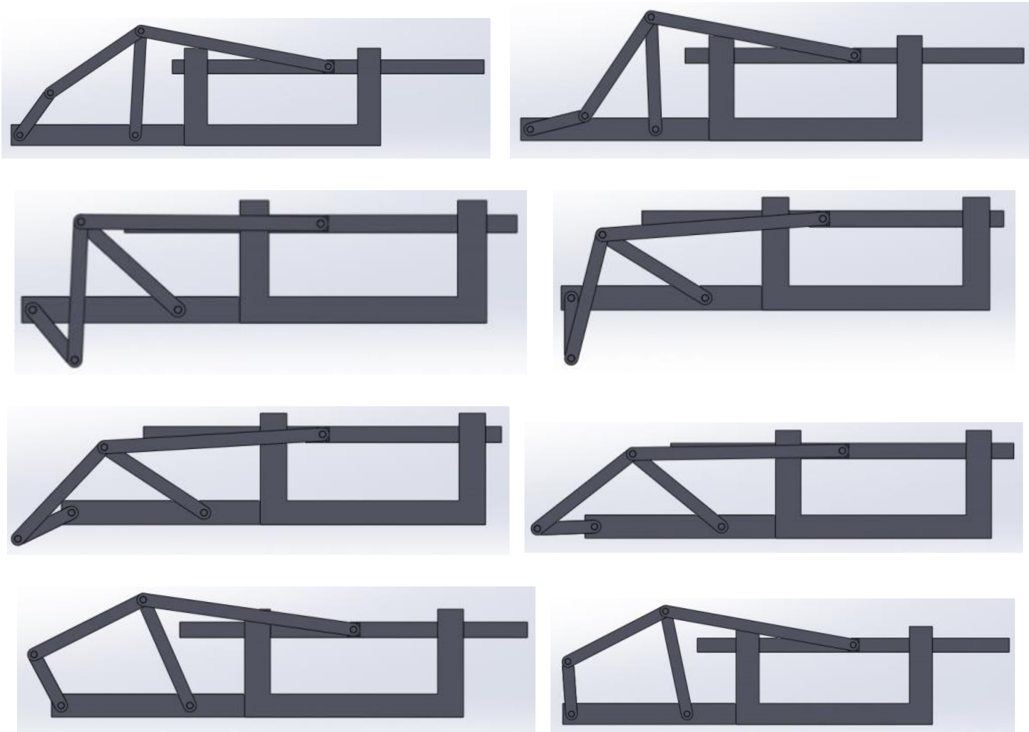


Fig. 7.18 Animation of the eight positions of the mechanism

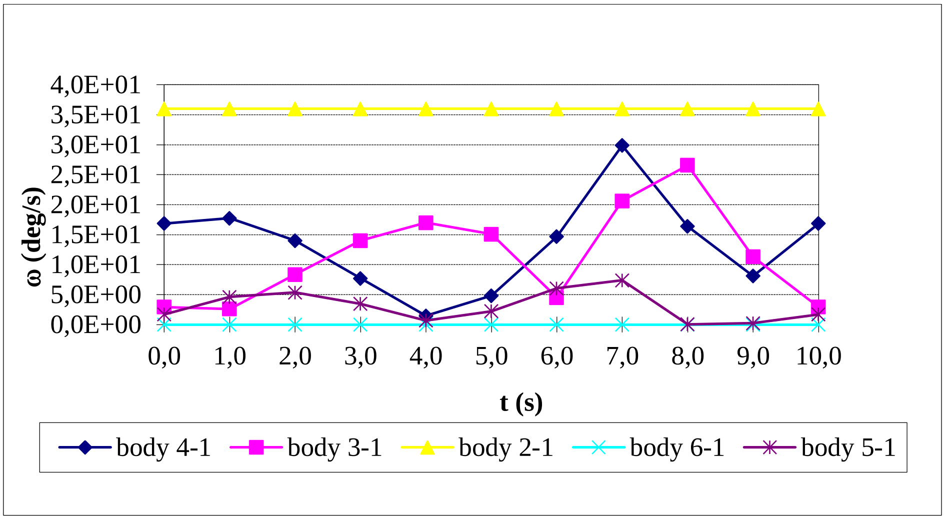


Fig. 7.19 Angular velocity of 2, 3, 4, 5, 6 bodies in dependence on the time

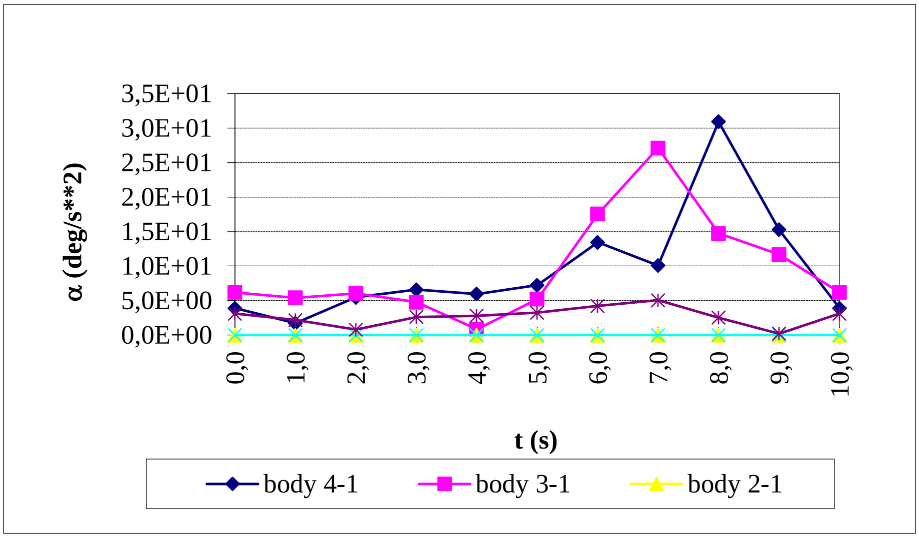


Fig. 7.20 Angular acceleration of 2, 3, 4, 5, 6 bodies in dependence on the time

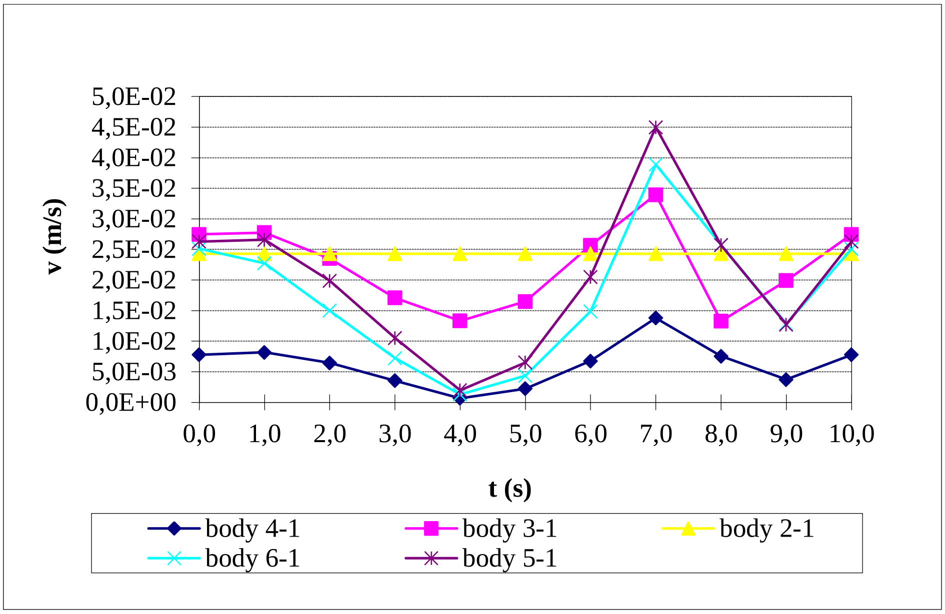


Fig. 7.21 Velocity or speed of 2, 3, 4, 5, 6 bodies in dependence on the time

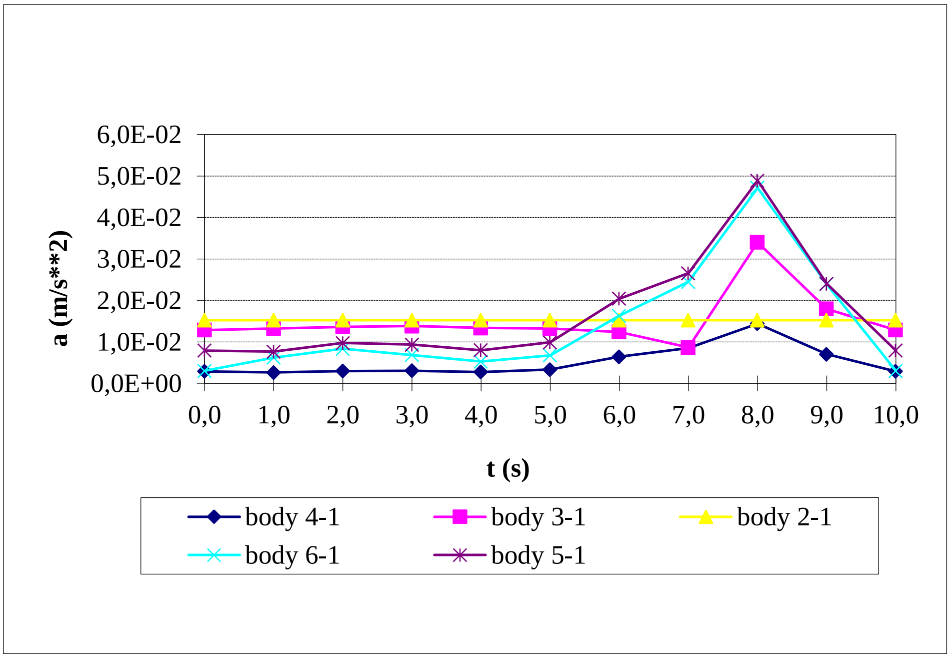


Fig. 7.22 Acceleration of bodies 2, 3, 4, 5, 6 – in dependence on the time

The results from the dynamic analysis:

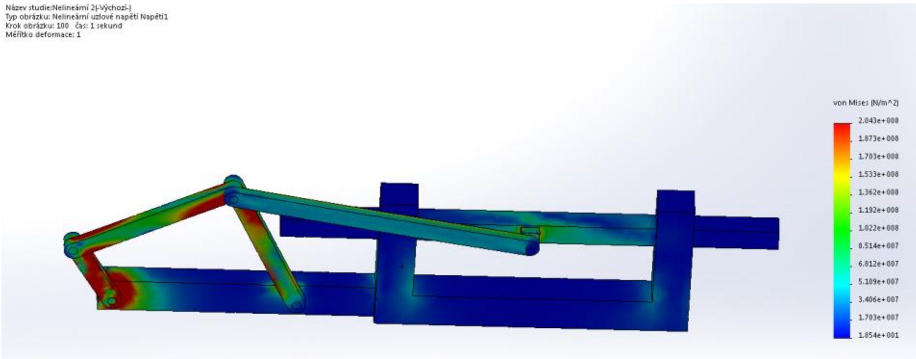


Fig. 7.23 Distribution of the stresses [Pa]

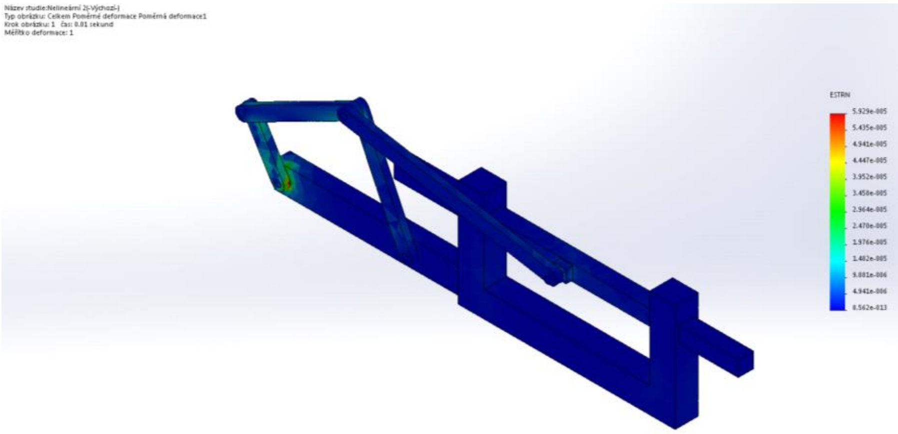


Fig. 7.24 Proportional deformation

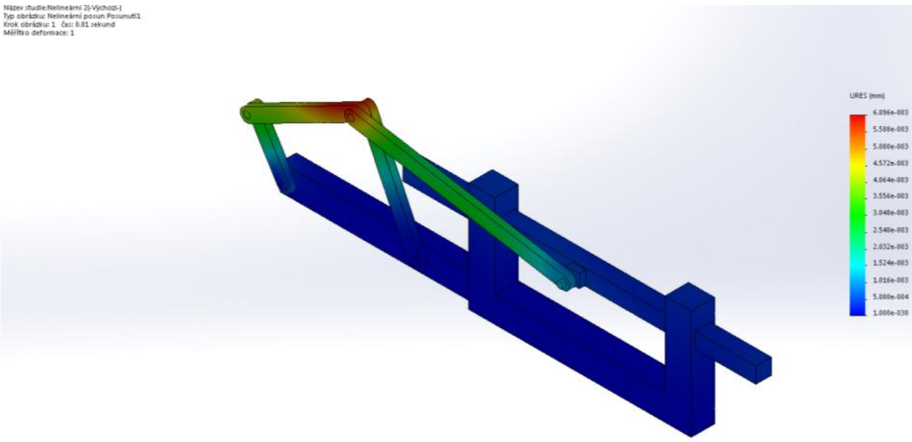


Fig. 7.25 Displacement course in [mm]

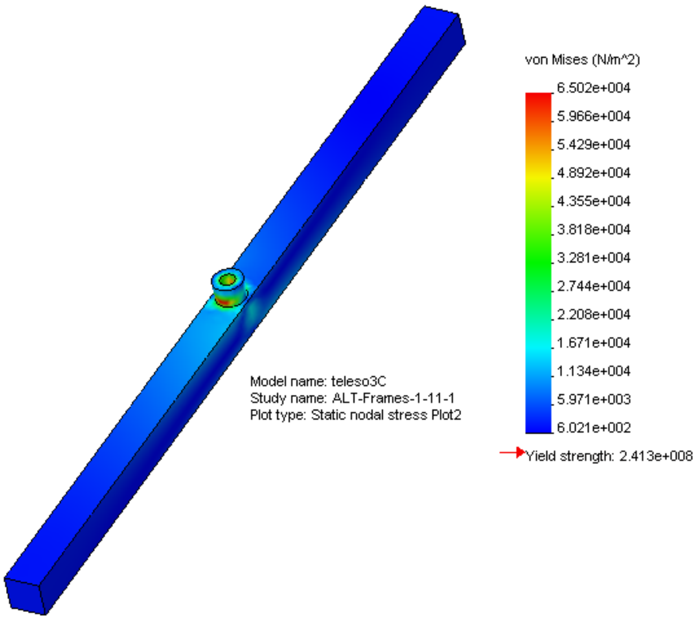


Fig. 7.26 Distribution of the stresses [Pa] for body, designated as 6

[Pa]

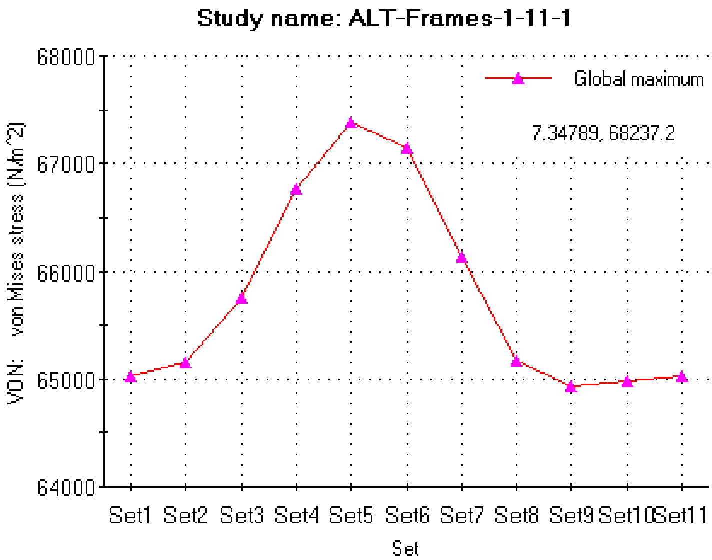


Fig. 7. 27 Course of the stresses for body, designated as 6 – in dependence on the time

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prof. Ing. Ján Vavro, CSc.

(born in 1952), accomplished the university study of the second degree in 1977 at the Mechanical Engineering Faculty of the University of Transport in Žilina, currently University of Žilina. In 1988, he was awarded a CSc. (equivalent: PhD.) degree at the Faculty of Mechanical Engineering of the University of Žilina. In 1995, he was qualified as a docent (lecturer professor) in the field of Applied Mechanics at the same faculty. In 2005, he was inaugurated at the Faculty of Mechanical Engineering at TU Košice in the field of Applied Mechanics, and in 2006, he was appointed a professor in the given mentioned scientific field. Since 1997, he has been working at the Faculty of Industrial Technologies in Púchov in the Department of Numerical Methods and Computational Modeling. He is the author of six monographs and co-author of 1 monograph, 2 university textbooks, 2 scripts and more than 355 scientific and professional works in domestic and foreign journals and different scientific collections. He solves grant projects (as a principal investigator as well as participating member in research team), research tasks, including tasks for practice. In the scientific and research field, he deals with static and dynamic analysis of mechanical systems and their optimization from the point of view of their material properties.

doc. Ing. Ján Vavro, PhD.

(born in 1982), accomplished the university study of the second degree in 2006 at Faculty of Industrial Technology in Púchov, Alexander Dubček University of Trenčín. In 2009, he accomplished the third degree of university study at Faculty of Industrial Technology in Púchov and he gained the PhD. title in the study field: 5.2.26 Materials. In 2014, he gained the scientific and pedagogical degree of Associate Professor (abbr: “Doc.”). Since 2009 he has been working in Department of Materials Engineering at Faculty of Industrial Technology in Púchov. He is the author of 3 monographs and co-author of 4 monographs, 2 university textbooks, 1 e-learning work and more than 115 scientific and professional works in national and international journals or proceedings. He is the investigator of grant projects, research issues as well as practical issues. In the field of science and research, he has been investigating the whole complex of problems relating to the identification of material defects by means of modal analysis. His scientific and research work is also devoted to the static and dynamic analysis of mechanical body systems and their optimisation in the terms of their material properties.

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